A STOCHASTIC MODEL OF EPIDEMIC

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First let's construct the determined model of the dynamic of uncontrolled epidemiological process. The quotient β describes frequency of meetings of sick people with healthy and probability of infection. It is subject to action of random factors. Let's enter an item for it which will take into account the influence of a random destabilization. We'll have stochastic model of epidemic. Comparing the determined and stochastic models, we'll find admissible borders for destabilizing quotient σ if the maximum deviation of dynamic variables can not be higher than 5 %.

Key words: The dynamic of epidemic, the determined uncontrolled model of epidemic, stochastic model of epidemic, destabilizing quotient, perturbed coefficient.

Determined model, describing uncontrolled process of the spread of epidemic, is described by a system of differential equations:

$$\begin{cases} \dot{x}(t) = -\beta x(t) y(t) - \mu x(t) + \Lambda, \\ \dot{y}_i(t) = \beta x(t) y(t) - (\mu + \tilde{\mu} + \gamma) y(t), \end{cases} \quad (1)$$

$$x(t) \ge 0,\ y(t) \ge 0,\ t \in [0,T],\ x(0) = x_0,\ y(0) = y_0, \quad (2)$$

where $\dot{x}(t)$ – the rate of change in the number of people exposed to the disease,

$\dot{y}_i((t)$ – the rate (speed) of change in the number of infected people,

$\beta x(t) y(t)$ – function characterizing the number of meetings of people exposed to the disease and infected ones per unit of time,

$\gamma y(t)$ – the number of people who regained their health per unit of time without the influence of external means: quarantine, vaccination and others (γ^{-1} average time of natural healing),

β – the growth coefficient, which characterizes the frequency of meetings of healthy people with infected people (in general case it can be considered as a function $\beta(x(t), y(t))$),

μ – the coefficient of natural mortality of people,

$\tilde{\mu}$ – the coefficient of mortality from this infection,

Λ – average birthrate (reproduction).

The considered mathematical model is determined and allows to calculate in advance the change of a condition of the studied system, on an interesting time segment by solving the Cauchy problem (1)–(2). We can assume that the values of some of the coefficients of the system in the moment $t \in [0,T]$ are not uniquely defined, for example, because of their dependence on many unpredictable factors, and they can be regarded as random processes, the mathematical expectations of which are known.

Assume that the coefficient of growth has a random component β, i.e. it can be represented as:

$$\beta(t) = m(t) + \sigma \cdot \xi(t, \omega), \quad (3)$$

where $m(t)$ – mathematical expectation of the coefficient β, set it permanent, i.e. $m(t)=\beta=const$; $\xi(t,\omega)$ – random process; σ – constant characterizing the degree of influence of the random perturbation on the value of the coefficient β.

In this case, the mathematical model (1)–(2) takes the following form:

$$\begin{cases} \dfrac{dx}{dt}=-\beta xy-\mu x+\Lambda-\sigma xy\xi(t,\omega), \\ \dfrac{dy}{dt}=\beta xy-(\mu+\tilde{\mu}+\gamma)y+\sigma xy\xi(t,\omega), \end{cases} \tag{4}$$

$$x(0,\omega)=x_0(\omega), \quad y(0,\omega)=y_0(\omega). \tag{5}$$

In this case, the state of the system $(x(t),y(t))$ is no longer a deterministic vector-function but is a vector random (stochastic) process $(x(t,\omega),y(t,\omega))$, $t\in[0,T]$.

In general, the system (4)–(5) can be written:

$$dX(t,\omega)=A(X,t)dt+B(X,t)df(t,\omega), \tag{6}$$

$$X(0,\omega)=X_0(\omega), \tag{7}$$

where $A:R^2\times[0,T]\to R^2$; $B:R^2\times[0,T]\to R^{2\times1}$; $f(t,\omega)$ – scalar Wiener process;

$$X(t)=\begin{pmatrix} x(t) \\ y(t) \end{pmatrix}\in R^2, \quad A(X,t)=\begin{pmatrix} -\beta xy-\mu x+\Lambda \\ \beta xy-(\mu+\tilde{\mu}+\gamma)y \end{pmatrix}\in R^2, \quad B(X,t)=\begin{pmatrix} -\sigma xy \\ \sigma xy \end{pmatrix}\in R^2, \quad \xi(t,\omega)\in R^1.$$

The obtained stochastic differential equation will be solved numerically, for this we use a stochastic analogue of Taylor's formula. Apply the unified stochastic Taylor-Ito expansion in iterated stochastic integrals and also the approximation of iterated stochastic integrals by means of the polynomial system of functions [1].

Formulate a theorem on Ito process expansion $\eta(s)=R(x(s),s)$, where R: $R^n\times[0,T]\to R^1$, in the unified Taylor-Ito series in iterated stochastic integrals $I^{l_1\ldots l_k}_{i_1\ldots i_k}(s,t)$.

Theorem 1. Let the process $\eta(s)=R(x(s),s)$ be Ito continuously differentiable $r+1$ times in the mean-square sense on $[0,T]$ along trajectories of the equation (6). Then for all $s,t\in[0,T]$, $s>t$ it decomposes into a unified Taylor-Ito series of the following type:

$$\eta(s)=\sum_{q=0}^{r}\left(\hat{C}^{D_q}\{\eta(\tau)\}\otimes(s\,\hat{\oplus}\,t)^{D_q}\right)+H_{r+1}(s,t) \tag{8}$$

and there exists such a constant $C_{r+1} < \infty$ that $\sqrt{M\{H_{r+1}(s,t)^2\}} \le C_{r+1}(s-t)^{\frac{(r+1)}{2}}$, $r = 0,1,...$, where

$$H_{r+1}(s,t) \overset{def}{=} \left(\hat{C}\{\eta(\tau)\}^{U_r} \otimes (s \hat{\oplus} t)^{U_r}\right) + D_{r+1}(s,t), \quad (9)$$

$$D_{r+1}(s,t) = \int_t^s \left(Q^{A_r}\{\eta(\tau)\}d\tau \otimes (s \oplus t)^{A_r}\right) + \int_t^s \left(\left(H^{A_r}\{\eta(\tau)\} \cdot df(\tau)^{1}\right) \otimes (s \oplus t)^{A_r}\right), \quad (10)$$

$$Q^{A_q}\{\eta(\tau)\} = \left\{L\,{}^{(k)}\hat{C}^{jl_1...l_k}\eta(\tau):\ (jl_1...l_k) \in A_q\right\},$$

$$H^{A_q}\{\eta(\tau)\} = \left\{{}^{(1)}G_0\,{}^{(k)}\hat{C}^{jl_1...l_k}\eta(\tau):\ (jl_1...l_k) \in A_q\right\},$$

$$\hat{C}^{D_q}\{\eta(\tau)\} = \left\{{}^{(k)}\hat{C}^{jl_1...l_k}\eta(\tau) : (jl_1...l_k) \in D_q\right\} \quad (11)$$

$${}^{(k)}\hat{C}^{jl_1...l_k}\{\cdot\} = \begin{cases} {}^{(k)}G_{l_1}...G_{l_k}L^j\{\cdot\} & if \ \ k > 0 \\ L^j\{\cdot\} & if \ \ k = 0 \end{cases},$$

$$L^j\{\cdot\} = \begin{cases} L...L\{\cdot\} & if \ \ j > 0 \\ & if \ \ j = 0 \end{cases},$$

$${}^{(1)}G_p\{\cdot\} = \frac{1}{p}\left({}^{(1)}G_{p-1}L\{\cdot\} - {}^{(1)}LG_{p-1}\{\cdot\}\right), \ \ p = 1,2,...,$$

$$L\{\cdot\} = \frac{\partial\{\cdot\}}{\partial t} + \sum_{i=1}^{n} a_i(x,t)\frac{\partial\{\cdot\}}{\partial x_i} + \frac{1}{2}\sum_{j=1}^{m}\sum_{l,i=1}^{n} b_{lj}(x,t)b_{ij}(x,t)\frac{\partial^2\{\cdot\}}{\partial x_l \partial x_i},$$

$$G_{0_i}\{\cdot\} = \sum_{j=1}^{n} b_{ji}(x,t)\frac{\partial\{\cdot\}}{\partial x_j}; \ \ i = 1,...,m$$

$$D_q = \left\{(k,j,l_1,...,l_k) : k + 2\left(j + \sum_{p=1}^{k} l_p\right) = q;\ k,j,l_1,...,l_k = 0,1,...\right\},$$

$$U_r = \left\{(k,j,l_1,...,l_k) : k + j + \sum_{p=1}^{k} l_p \le r\right\},$$

$$A_q = \left\{(k,j,l_1,...,l_k) : k + j + \sum_{p=1}^{k} l_p = q;\ k,j,l_1,...,l_k = 0,1,...\right\},$$

equality (8) is just (is fair, is true, takes place, holds) with probability 1, right parts of (8)–(10) exist in the mean-square sense.

We construct a unified Taylor-Ito expansion for the components of the solution $X(t)$ of the system (4)–(5) for infinitesimal of order of $O((s-t)^{5/2})$, i.e. we will construct expansion of Ito process $\eta(t)=X(t)$:

$$
\begin{aligned}
x(s) &= x(t)+(s-t)[\left(-\beta xy-\mu x+\Lambda\right)+\sigma xy(\beta(y-x)+\mu)I_1^0(s,t)+\sigma^2 xy(\beta(2xy-(x-y)^2)\\
&+\mu(x-y))I_{11}^{00}(s,t)]\ +\frac{(s-t)^2}{2}\left(\beta^2 xy(y-x)+2\beta\mu xy+\mu^2 x-\Lambda\beta y-\Lambda\mu+(\mu+\tilde{\mu}+\gamma)\beta xy\right)\\
&-\sigma x\,y\,I_1^0(s,t)+\sigma^2 x\,y\,(y-x)I_{11}^{00}(s,t)+\sigma^3 xy(2xy-(x-y)^2)I_{111}^{000}(s,t)\\
&+\sigma y(\Lambda-(\mu+\tilde{\mu}+\gamma)x)I_1^1(s,t)+\sigma^4 x\,y\,(11x\,y\,(x-y)-(x^3-y^3))I_{1111}^{0000}(s,t)\\
&-\sigma^2 y\,(\Lambda(y-x)-x\,y\,(\mu+\tilde{\mu}+\gamma)+\mu x^2)I_{11}^{10}(s,t)+\sigma^2 x\,y\,((\mu+\tilde{\mu}+\gamma)(y-1)+\Lambda)I_{11}^{01}(s,t)\ +H_5^x(s,t),
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
y(s) &= y(t)+(s-t)[\left(\beta xy-(\mu+\tilde{\mu}+\gamma)y\right)+\sigma xy(\beta(x-y)-(\mu+\tilde{\mu}+\gamma))I_1^0(s,t)\\
&+\sigma^2 xy(\beta(-2xy+(x-y)^2)+(\mu+\tilde{\mu}+\gamma)(y-x))I_{11}^{00}(s,t)]\\
&+\frac{(s-t)^2}{2}\left(\beta^2 xy(x-y)-2\beta xy(\mu+\tilde{\mu}+\gamma)-\beta\mu xy+\Lambda\beta y+(\mu+\tilde{\mu}+\gamma)^2 y\right)\\
&+\sigma\,xyI_1^0(s,t)+\sigma^2 xy(x-y)I_{11}^{00}(s,t)+\sigma^3 xy(2xy-(x-y)^2)I_{111}^{000}(s,t)+\sigma y(\mu x-\Lambda)I_1^1(s,t)\\
&-\sigma^4 x\,y\,(11xy\,(x-y)-(x^3-y^3))I_{1111}^{0000}(s,t)+\sigma^2 y\,(\Lambda(y-x)-xy(\mu+\tilde{\mu}+\gamma)+\mu x^2)I_{11}^{10}(s,t)\\
&+\sigma^2 xy\,(\mu\,(x-y)-\Lambda)I_{11}^{01}(s,t)+H_5^y(s,t),
\end{aligned}
\tag{13}
$$

Relations (12)–(13) on a uniform discrete grid $\{\tau_j\}_{j=0}^N$ constructed for the segment $[0,T]$, such that $\tau_j = j\Delta$, $\tau_N = N\Delta = T$ are selected as a numerical method for modeling the system (6)–(7). Denote $x(\tau_j)=x_j$, $y(\tau_j)=y_j$, and then by putting $s=(k+1)\Delta$, $t=k\Delta$, $k=0,1,\ldots$ in expansions (12)–(13) and using the expansions of the iterated stochastic integrals $I_1^0(s,t), I_1^1(s,t),\ldots$ in terms of a polynomial basis, the following expressions for the numerical method are obtained:

$$
\begin{aligned}
x_{k+1} &= x_k+\Delta[\left(-\beta xy-\mu x+\Lambda\right)+\sigma xy(\beta(y-x)+\mu)I_1^0(s,t)+\sigma^2 xy(\beta(2xy-(x-y)^2)\\
&+\mu(x-y))I_{11}^{00}(s,t)]+\frac{\Delta^2}{2}\left(\beta^2 xy(y-x)+2\beta\mu xy+\mu^2 x-\Lambda\beta y-\Lambda\mu+(\mu+\tilde{\mu}+\gamma)\beta xy\right)\\
&-\sigma xyI_1^0(s,t)+\sigma^2 xy(y-x)I_{11}^{00}(s,t)+\sigma^3 xy(2xy-(x-y)^2)I_{111}^{000}(s,t)+\sigma y(\Lambda-(\mu+\tilde{\mu}\\
&+\gamma)x)I_1^1(s,t)+\sigma^4 xy(11xy(x-y)-(x^3-y^3))I_{1111}^{0000}(s,t)\\
&-\sigma^2 y(\Lambda(y-x)-xy(\mu+\tilde{\mu}+\gamma)+\mu x^2)I_{11}^{10}(s,t)+\sigma^2 xy((\mu+\tilde{\mu}+\gamma)(y-1)+\Lambda)I_{11}^{01}(s,t),
\end{aligned}
\tag{14}
$$

$$
\begin{aligned}
y_{k+1} &= y_k + \Delta[\left(\beta xy - (\mu+\tilde{\mu}+\gamma)y\right) + \sigma xy(\beta(x-y) - (\mu+\tilde{\mu}+\gamma))I_1^0(s,t) \\
&+\sigma^2 xy(\beta(-2xy+(x-y)^2) + (\mu+\tilde{\mu}+\gamma)(y-x))I_{11}^{00}(s,t)] + \frac{\Delta^2}{2}[\beta^2 xy(x-y) \\
&-2\beta xy(\mu+\tilde{\mu}+\gamma) - \beta\mu xy + \Lambda\beta y + (\mu+\tilde{\mu}+\gamma)^2 y] + \sigma xy I_1^0(s,t) \\
&+\sigma^2 xy(x-y)I_{11}^{00}(s,t) + \sigma^3 xy(2xy-(x-y)^2)I_{111}^{000}(s,t) + \sigma y(\mu x - \Lambda)I_1^1(s,t) \\
&-\sigma^4 xy(11xy(x-y) - (x^3-y^3))I_{1111}^{0000}(s,t) \\
&+\sigma^2 y(\Lambda(y-x) - xy(\mu+\tilde{\mu}+\gamma) + \mu x^2)I_{11}^{10}(s,t) + \sigma^2 xy(\mu(x-y)-\Lambda)I_{11}^{01}(s,t),
\end{aligned} \tag{15}
$$

where

$$
\begin{aligned}
I_1^0(\tau_{k+1},\tau_k) &= \sqrt{\Delta}\varsigma_0^{(1)}, \\
I_1^1(\tau_{k+1},\tau_k) &= -\frac{\Delta^{3/2}}{2}\left[\varsigma_0^{(1)} + \frac{1}{\sqrt{3}}\varsigma_1^{(1)}\right], \\
I_{11}^{00}(\tau_{k+1},\tau_k) &= \frac{\Delta}{2}\left[\varsigma_0^{(1)}\cdot\varsigma_0^{(1)} + \frac{1}{\sqrt{3}}(\varsigma_0^{(1)}\varsigma_1^{(1)} - \varsigma_1^{(1)}\varsigma_0^{(1)})\right] = \frac{\Delta}{2}\cdot[(\varsigma_0^{(1)})^2 - 1], \\
I_{111}^{000}(\tau_{k+1},\tau_k) &= \frac{\Delta^{3/2}}{6}\left[\left(\varsigma_0^{(1)}\right)^3 - 3\varsigma_0^{(1)}\right], \\
I_{1111}^{0000}(\tau_{k+1},\tau_k) &= \frac{\Delta^2}{24}\left[\left(\varsigma_0^{(1)}\right)^4 - 6\left(\varsigma_0^{(1)}\right)^2 + 3\right], \\
I_{11}^{01}(\tau_{k+1},\tau_k) &= -\frac{\Delta^2}{4}[\frac{4}{3}(\varsigma_0^{(1)})^2 + \frac{1}{\sqrt{3}}\varsigma_0^{(1)}\cdot\varsigma_1^{(1)} + \frac{1}{3\sqrt{5}}\varsigma_0^{(1)}\cdot\varsigma_2^{(1)} \\
&+\sum_{i=1}^{q}\left\{\frac{1}{\sqrt{(2i+1)(2i+5)}(2i+3)}\varsigma_i^{(1)}\cdot\varsigma_{i+2}^{(1)} - \frac{1}{(2i-1)(2i+3)}(\varsigma_i^{(1)})^2\right\} - 2
\end{aligned} \tag{16}
$$

$$
\begin{aligned}
I_{11}^{10}(\tau_{k+1},\tau_k) &= -\frac{\Delta^2}{2}\varsigma_0^{(1)}(\varsigma_0^{(1)} + \frac{1}{\sqrt{3}}\varsigma_1^{(1)}) - I_{11}^{01}(\tau_{k+1},\tau_k) = -\frac{\Delta^2}{4}[\frac{2}{3}(\varsigma_0^{(1)})^2 + \frac{1}{\sqrt{3}}\varsigma_0^{(1)}\cdot\varsigma_1^{(1)} \\
&-\frac{1}{3\sqrt{5}}\varsigma_0^{(1)}\cdot\varsigma_2^{(1)} + \sum_{i=1}^{q}\left\{-\frac{1}{\sqrt{(2i+1)(2i+5)}(2i+3)}\varsigma_i^{(1)}\cdot\varsigma_{i+2}^{(1)} + \frac{1}{(2i-1)(2i+3)}(\varsigma_i^{(1)})^2\right\} + 2]
\end{aligned}
$$

$\{\varsigma_i^{(j)},\ i=0,1,...,q+2;\ j=1\}$ – a system of independent Gaussian random variables with zero mean (expectation) and variance of one, which is generated on a step of integration with a number of k and is independent with the analogous systems of random variables that are generated on all the preceding steps of integration towards (with respect to) the step of integration with the number of k; Δ – step of integration of the numerical method; the number q is chosen from the condition ([1], p. 199):

$$
\begin{aligned}
&M\left\{\left(I_{11}^{10}(\tau_{k+1},\tau_k) - I_{(11)q}^{10}(\tau_{k+1},\tau_k)\right)^2\right\} = M\left\{\left(I_{11}^{01}(\tau_{k+1},\tau_k) - I_{(11)q}^{01}(\tau_{k+1},\tau_k)\right)^2\right\} \\
&\le \frac{\Delta^4}{16}\left(\frac{3}{16}(\frac{\pi^4}{90} - \sum_{i=1}^{q}\frac{1}{i^4}) + (\frac{1}{2} - \sum_{i=1}^{q}\frac{1}{4i^2-1})^2\right) \le C\Delta^5,
\end{aligned} \tag{17}
$$

where constant C must be given. We choose it for the sake of simplicity to be unity (to be equal to one). The value of q increases with a decrease in the value of the step of integration. Consider the results of the choice of number q with the help of the relation (11). These results are placed in the following table.

Δ	0,004	0,001	0,0005
Q	1	2	4

I.e., it is enough to let q be equal to 1 that Δ would be 0,004, then the expansions for I_{11}^{10} and I_{11}^{01} take the form:

$$I_{11}^{01}(\tau_{k+1},\tau_k)=-\frac{\Delta^2}{4}[\frac{4}{3}(\varsigma_0^{(1)})^2+\frac{1}{\sqrt{3}}\varsigma_0^{(1)}\cdot\varsigma_1^{(1)}+\frac{1}{3\sqrt{5}}\varsigma_0^{(1)}\cdot\varsigma_2^{(1)}+\frac{1}{5\sqrt{21}}\varsigma_1^{(1)}\cdot\varsigma_3^{(1)}-\frac{1}{5}(\varsigma_1^{(1)})^2-2],$$

$$I_{11}^{10}(\tau_{k+1},\tau_k)=-\frac{\Delta^2}{4}[\frac{2}{3}(\varsigma_0^{(1)})^2+\frac{1}{\sqrt{3}}\varsigma_0^{(1)}\cdot\varsigma_1^{(1)}-\frac{1}{3\sqrt{5}}\varsigma_0^{(1)}\cdot\varsigma_2^{(1)}-\frac{1}{5\sqrt{21}}\varsigma_1^{(1)}\cdot\varsigma_3^{(1)}+\frac{1}{5}(\varsigma_1^{(1)})^2+2].$$

Make the numerical modeling of the solution of the system (6)–(7) by means of relations (14)–(15) on the time interval $T = 10$ with the step $\Delta = 0{,}004$ with the following input (initial) data: $\beta = 2\cdot10^{-6}$; $\mu = 0{,}003$; $\tilde{\mu} = 0$; $\Lambda = 20$; $\gamma = 1$; $X_0(\omega) =$ 380 000; $Y_0(\omega) = 2000$; $\sigma = 0$. The result of the numerical modeling is presented on fig. 1. Now introduce the stochastic perturbation $\sigma > 0$. The evolution of processes $x(t,\omega), y(t,\omega)$, which characterize the process of evolution of epidemic of the system (6)-(7) for values $\sigma = 10^{-7}; 2\cdot10^{-7}; 5\cdot10^{-7}; 7\cdot10^{-7}; 10^{-6}$ is presented on figs. 2–6, respectively. The values of the maximal (maximum) trajectories deviations of the perturbed system from the trajectories of the determined system are listed in table 1, from which direct dependence of maximal (maximum) deviations of the solution of a perturbed system on the value of the perturbed parameter σ is well seen.

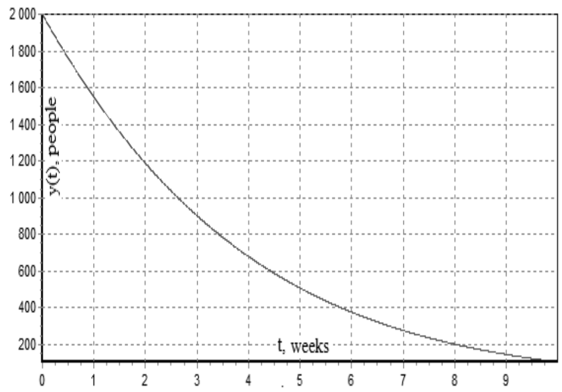

Fig. 1. Determined model of epidemic, $\sigma = 0$

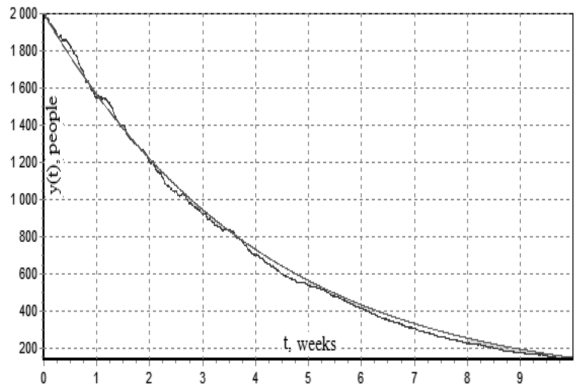

Fig. 2. Determined and stochastic models of epidemic, $\sigma = 10^{-7}$

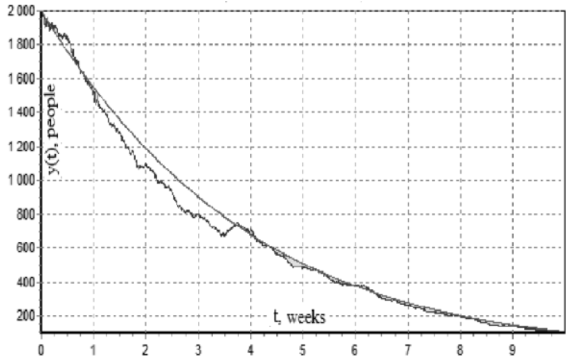

Fig. 3. $\sigma = 1{,}5 \cdot 10^{-7}$

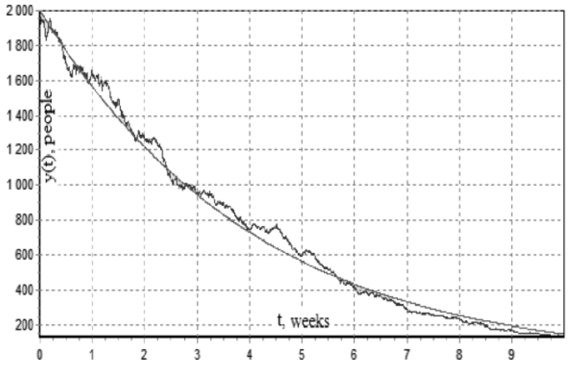

Fig. 4. $\sigma = 2 \cdot 10^{-7}$

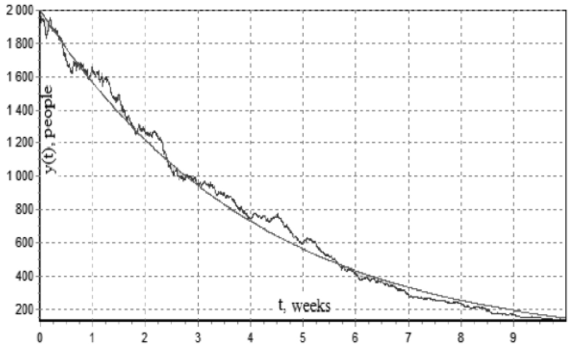

Fig. 5. $\sigma = 2{,}5 \cdot 10^{-7}$

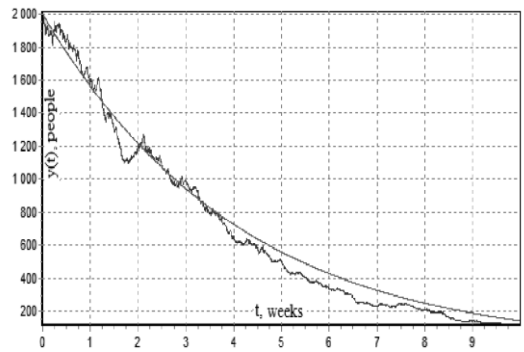

Fig. 6. $\sigma = 3 \cdot 10^{-7}$

Table 1

Value of σ	Maximum deviation of the trajectories of the perturbed system from the trajectories of the determined system depending on the value of the parameter σ	
	Deviation X, %	Deviation Y, %
10^{-7}	Practically none	2,5
$1{,}5 \cdot 10^{-7}$	Practically none	3,5
$2 \cdot 10^{-7}$	0,1	4
$2{,}5 \cdot 10^{-7}$	0,12	4,5
$3 \cdot 10^{-7}$	0,15	5

For $\sigma \le 10^{-8}$ stochastic model is almost equal to the determined one, so it is necessary to take the determined model for description of the system; for $\sigma > 3 \cdot 10^{-7}$ the stochastic model significantly (by more than 5%) is different from the determined one; therefore the perturbed coefficient needs to be taken ranging from 10^{-8} to $3 \cdot 10^{-7}$.

For different realizations of the system of independent Gaussian values $\left\{ \varsigma_i^{(j)},\ i = 0,1,...,q+2;\ j = 1,2 \right\}$ we obtain different realizations of the solution of the system of stochastic differential equations (6). These trajectories for small perturbations lie inside of a tube constructed in a small neighbourhood of the solution of determined system (1). Find mean of the solution of system (6) in 5, 10 realizations for $\sigma = 5 \cdot 10^{-7}$, in 5, 10, 15 and 50 realizations for $\sigma = 10^{-6}$.

On figs. 7–12 it is shown a comparison of means with the solution of the determined system of differential equations (1).

One can conclude that $M\{X(t;\omega)\} \to X(t)$, which is confirmed by numerical experiments, where $X(t) = (x(t), y(t))$ – solution of determined system (1), (2), $X(t;\omega) = (x(t,\omega), y(t,\omega))$ – solution of stochastic system (4), (5).

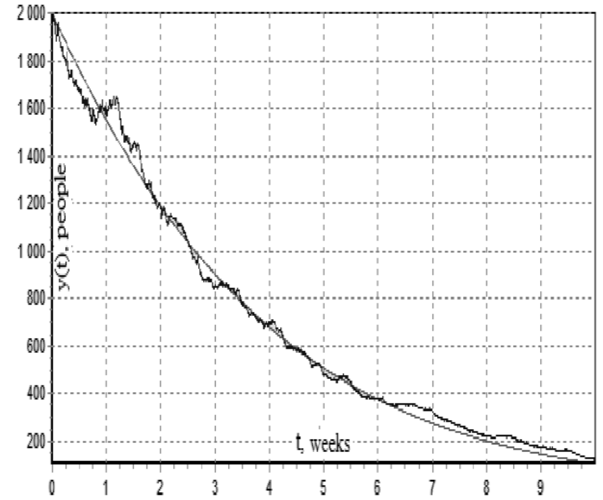

Fig. 7. Determined model and mean of the solution of system (6, 7) found in 5 realizations ($\sigma = 5 \cdot 10^{-7}$)

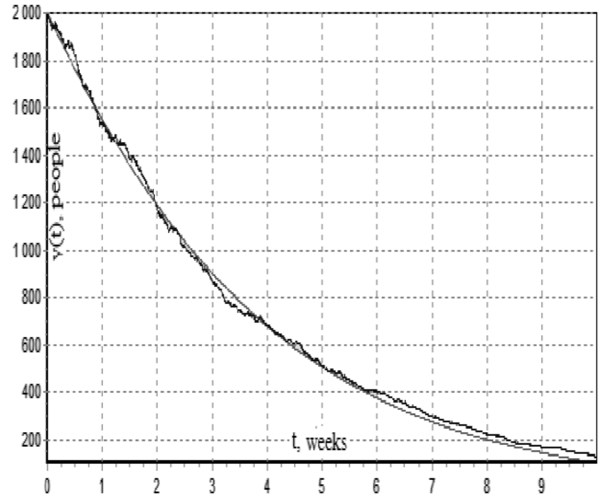

Fig. 8. Determined model and mean of the solution of system (6, 7) found in 10 realizations ($\sigma = 5 \cdot 10^{-7}$)

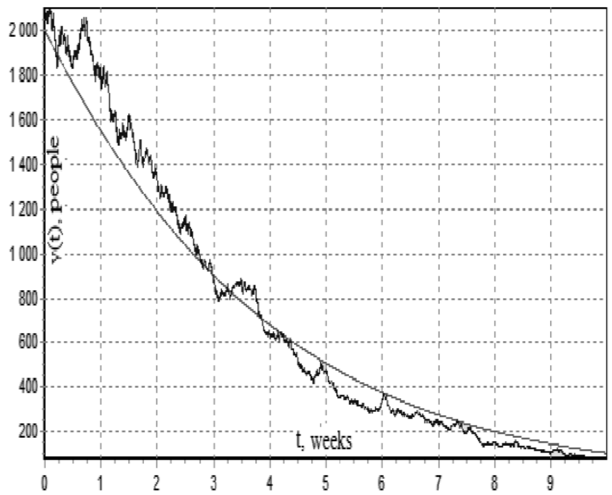

Fig. 9. Determined model and mean of the solution of system (6, 7) found in 5 realizations ($\sigma = 10^{-6}$)

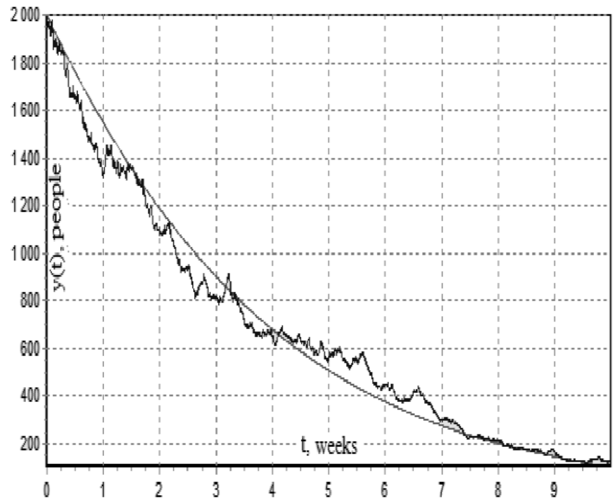

Fig. 10. Determined model and mean of the solution of system (6, 7) found in 10 realizations ($\sigma = 10^{-6}$)

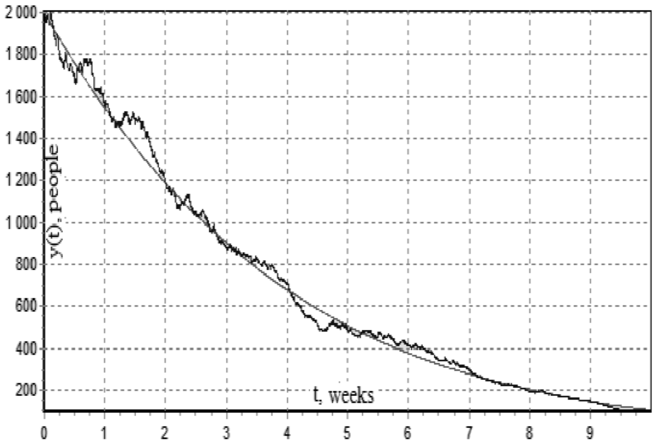

Fig. 11. Determined model and mean of the solution

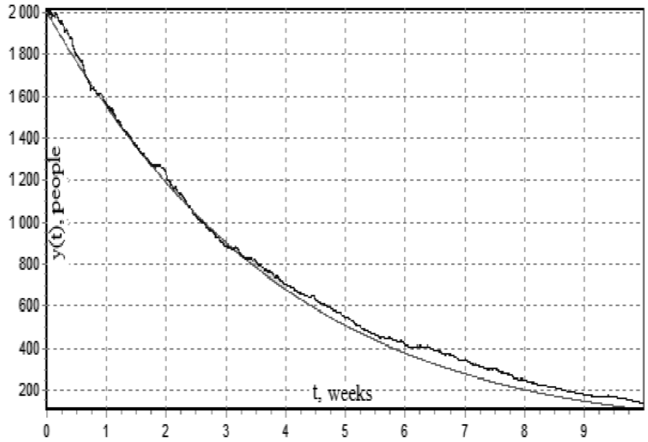

Fig. 12. Determined model and mean of the solution

of system (6, 7) found in 15 realizations ($\sigma = 10^{-6}$) of system (6, 7) found in 50 realizations ($\sigma = 10^{-6}$)

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СТОХАСТИЧЕСКАЯ МОДЕЛЬ ДИНАМИКИ ЭПИДЕМИИ

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Строится детерминированная модель динамики неуправляемого эпидемиологического процесса. Затем, считая, что коэффициент β, характеризующий частоту встреч и вероятность заражения при встрече, подвержен воздействию случайных факторов, введем для него слагаемое, учитывающее влияние случайного возмущения. Получим стохастическую модель эпидемии. Сравнивая детерминированную и стохастическую модели, найдем допустимые границы для возмущенного коэффициента σ при условии, что максимальное отклонение динамических переменных не должно превышать 5 %.

Ключевые слова: Динамика эпидемии, детерминированная неконтролируемая модель эпидемии, стохастическая модель эпидемии, дестабилизирующий фактор, возмущенный коэффициент.

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