CONTACT INTEGRABLE EXTENSIONS OF SYMMETRY PSEUDO-GROUP AND COVERINGS FOR THE R-th DOUBLE MODIFIED DISPERSIONLESS KADOMTSEV-PETVIASHVILI EQUATION[1]

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We find contact integrable extensions and coverings for the r-th double modified dispersionless Kadomtsev-Petviashvili law equation. One of the coverings provides a Bäcklund auto-transformation and a recursion operator for the equation under the study.

Key words: Lie pseudo-group, differential covering, contact integrable extension, Bäcklund transformation.

We consider the r-th double modified dispersionless Kadomtsev-Petviashvili law equation

$$u_{yy} = u_{tx} + \left(\frac{(\kappa+1)\,u_y^2}{u_x^2} - \frac{u_t}{u_x} + \kappa\,u_x^{\kappa} u_y + \frac{(\kappa+1)^2}{2\,\kappa+3}\,u_x^{2(\kappa+1)}\right) u_{xx} - \kappa\,\left(\frac{u_y}{u_x} + u_x^{\kappa+1}\right) u_{xy} \tag{1}$$

with $\kappa \notin \{-2, -3/2, -1\}$. This equation arises from the differential covering [5, 6, 7],

$$\begin{cases} u_t = \left(\dfrac{(\kappa+2)^2}{2\,\kappa+3}\,u_x^{2(\kappa+1)} + (\kappa+2)\,w_x\,u_x^{\kappa+1} + \dfrac{\kappa+1}{2}\,w_x^2 - w_y\right) u_x \\ u_y = -(u_x^{\kappa+1} + w_x)u_x \end{cases} \tag{2}$$

over the r-th modified dispersionless Kadomtsev-Petviashvili law equation [1], written in the form

$$w_{yy} = w_{tx} + \left(\frac{1}{2}\,(\kappa+1)\,w_x^2 + w_y\right) w_{xx} + \kappa\,w_x\,w_{xy}, \tag{3}$$

with $r = \kappa\,(\kappa+1)^{-1}$, see [2, 4, 14, 12]. Namely, excluding w from (2) yields Eq. (1).

We apply the method of contact integrable extensions [11], to find differential coverings of Eq. (1). The method starts from computing Maurer-Cartan law forms and structure equations for the symmetry pseudo-group by the methods described in [13, 3, 9, 10]. The structure equations are given in Appendix. These equations are not involutive. Their involutive completion includes equations for differentials of forms $\eta_1, \dots, \eta_7$. The completion may be obtained by the normal prolongation [16, 13, 15], of system (23). Those equations are too big to be written in full here.

The Maurer-Cartan law forms $\theta_0, \dots, \theta_{23}, \xi^1, \xi^2, \xi^3$ are

$$\theta_0 = u_{xx}u_x^{-2}\,(du - u_t\,dt - u_x\,dx - u_y\,dy),$$

$$\theta_1 = u_x^{-2\kappa-3}\,(du_t - u_{tt}\,dt - u_{tx}\,dx - u_{ty}\,dy) - (\kappa+2)\,(u_y\,u_x^{-\kappa-2} - 1)\,\theta_3 + ((\kappa+1)$$
$$(\kappa+2)\,(u_y u_x^{-\kappa-2} - (2\kappa+3)^{-1}) - u_t u_x^{-2\kappa-3}))\,\theta_2 + (u_t u_x^{-2\kappa-3} + (\kappa+1)^2(\kappa+2)$$
$$(u_y u_x^{-\kappa-2} - (2\kappa+5)(2\kappa+3)^{-1})\,\theta_0$$

$$\theta_2 = u_x^{-1}\,(du_x - u_{tx}\,dt - u_{xx}\,dx - u_{xy}\,dy),$$

$$\theta_3 = u_x^{-\kappa-2}\,\left(du_y - u_{tx}\,dt - u_{xy}\,dx - E\,dy\right) - \left(u_y\,u_x^{-\kappa-3} - \kappa - 1\right)\theta_2$$
$$-(u_y\,u_x^{-\kappa-3} + (\kappa+1)^2)\,\theta_0,$$

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$$\theta_{11} = u_{xx}^{-1}u_x^{-4\kappa-4}\,(du_{tt} - u_{ttt}\,dt - u_{ttx}\,dx - u_{tty}\,dy) - 2\,(\kappa+2)(u_y u_x^{-\kappa-2} - 1)\,\theta_{13}$$
$$-(2\,u_t u_x^{-2\kappa-3} - (\kappa+2)((\kappa+2)\,u_y^2 u_x^{-2\kappa-4} - (2\kappa+3)\,u_y u_x^{-\kappa-2}$$
$$+(2\kappa^2+9\kappa+8)(2\kappa+3)^{-1}))\,\theta_{12} + A_{110}\,\theta_0 + A_{111}\,\theta_1 + A_{112}\,\theta_2 + A_{113}\,\theta_3$$
$$-(u_t^2 u_x^{-4\kappa-6} + (\kappa+1)^2(\kappa+2)^2\,(u_y u_x^{-\kappa-2} - (2\kappa+3)^{-1})^2$$
$$+2\,(\kappa+1)(\kappa+2)(2\kappa+3)^{-1}\,u_x^{-2\kappa-3})\,\theta_{22} - 2\,(\kappa+2)\,((u_y u_x^{-\kappa-2} - 1)u_t u_x^{-2\kappa-3}$$
$$-(\kappa+1)(\kappa+2)(u_y^2 u_x^{-2\kappa-4} + 2\,(\kappa+2)(2\kappa+3)^{-3}\,u_x^{-\kappa-2} - 2\kappa - 3))\,\theta_{23},$$
$$\theta_{12} = u_{xx}^{-1}u_x^{-2\kappa-2}\,(du_{tx} - u_{ttx}\,dt - u_{txx}\,dx - u_{txy}\,dy) - (u_y u_x^{-\kappa-2} - 1)\,\theta_{23}$$
$$+((\kappa+1)(\kappa+2)(u_y u_x^{-\kappa-2} - (2\kappa+3)^{-1}) - u_t u_x^{-2\kappa-3})\,(\theta_{22} + \theta_3) - \theta_1$$
$$-(u_{txx}u_{xx}^{-2}u_x^{-2\kappa-1} + 2\,u_{tx}u_{xx}^{-1}u_x^{-2\kappa-2} - \frac{1}{2}\,(\kappa+2)\,(u_{xy}u_{xx}^{-1}\,((\kappa+2)u_y u_x^{-2\kappa-3} + \kappa\,u_x^{-\kappa-1})$$
$$-(\kappa+2)\,(u_y^2 u_x^{-2\kappa-4} - (\kappa+1)\,u_y u_x^{-\kappa-2}) + \kappa\,(\kappa+1)))\,\theta_0,$$
$$\theta_{13} = u_{xx}^{-1}u_x^{-3\kappa-3}\,(du_{ty} - u_{tty}\,dt - u_{txy}\,dx - \bar{\mathbb{D}}_t(E)\,dy) - (2\kappa+3)(u_y u_x^{-\kappa-2} - 1)\,\theta_{12}$$
$$-(u_y u_x^{-\kappa-2} + (\kappa+1)^2)\,\theta_1 - ((u_y u_x^{-3\kappa-5} - (\kappa+1)\,u_x^{-2\kappa-3})\,u_t - (\kappa+1)\,(u_y^2 u_x^{-2\kappa-4}$$
$$-(\kappa+2)(2\kappa+3)^{-1}((2\kappa^2+5\kappa+4)u_y u_x^{-\kappa-2} - \kappa - 1))\,\theta_{22} + A_{130}\,\theta_0 + A_{132}\,\theta_2$$
$$+A_{133}\,\theta_3 - (u_t u_x^{-2\kappa-3} - (\kappa+2)\,(u_y^2 u_x^{-2\kappa-4} - (2\kappa+3)\,u_y u_x^{-\kappa-2}$$
$$+2\,(\kappa+1)(\kappa+2)(2\kappa+3)^{-1}))\,\theta_{23},$$
$$\theta_{22} = u_{xx}^{-1}\,(du_{xx} - u_{txx}\,dt - u_{xxx}\,dx - u_{xxy}\,dy) - 2\,\theta_2 - u_x u_{xxx} u_{xx}^{-2}\,\theta_0,$$
$$\theta_{23} = u_x^{-\kappa-1}u_{xx}^{-1}\,(du_{xy} - u_{txy}\,dt - u_{xxy}\,dx - \bar{\mathbb{D}}_x(E)\,dy) - (u_y u_x^{-\kappa-2} - \kappa - 1)\,\theta_{22} - \theta_3$$
$$+\frac{1}{2}\,(\kappa\,u_{xy}u_{xx}^{-1}u_x^{-\kappa-1} - (\kappa+4)\,u_y u_x^{-\kappa-2} - \kappa\,(\kappa+1))\,\theta_2$$
$$-(u_{xxy}u_{xx}^{-2}\,u_x^{-\kappa} - u_{xy}u_{xx}^{-1}\,u_x^{-\kappa-1} + u_y u_x^{-\kappa-2} + (\kappa+1)^2)\,\theta_0,$$
$$\xi^1 = u_{xx}u_x^{2\kappa+1}\,dt,$$
$$\xi^2 = u_{xx}u_x^{-1}\,dx + (u_t u_x^{-\kappa-3} + (\kappa+2)\,(u_y^2 u_x^{-2\kappa-4} - u_y u_x^{-2} + 2\,(\kappa+1)^2(2\kappa+3)^{-1}))\,\xi^1$$
$$+(u_y u_x^{\kappa-2} - \kappa - 1)\,\xi^3,$$
$$\xi^3 = u_{xx}u_x^{\kappa}\,dy + (\kappa+2)\,(u_y u_x^{-\kappa-2} - 1)\,\xi^1, \quad (4)$$

where E is the right-hand side of Eq. (1), $\bar{\mathbb{D}}_t$, $\bar{\mathbb{D}}_x$ are restrictions of the total derivatives on Eq. (1), and A_{110}, A_{111}, A_{112}, A_{113}, A_{130}, A_{132}, A_{133} are functions of derivatives of u of the first and the second orders. These functions are too long to write them in full. The coefficients of the structure equations depend on the invariants

$$U_1 = (\kappa+2)\,\big(u_y u_x^{-\kappa-2} - u_{xy}u_{xx}^{-1}u_x^{-\kappa-1} + \kappa + 1\big),$$
$$U_2 = u_{txx}u_{xx}^{-2}u_x^{-2\kappa-1} - (\kappa+2)\,u_{xxy}u_{xx}^{-2}u_x^{-\kappa}\big(u_y u_x^{-\kappa-2} - 1\big) - 2\,u_{tx}u_{xx}^{-2}u_x^{-2\kappa-2} + 2\,u_t u_x^{-2\kappa-3}$$
$$-\Big(2u_y u_x^{-\kappa-2} - (\kappa+1)(\kappa+2)\Big)\,U_1 + 2\,(\kappa+1)(\kappa+2)\,u_y u_x^{-\kappa-2}$$

$u_{xxx}u_{xx}^{-2}\,(u_t u_x^{-2\kappa-2} - (\kappa+2)u_y u_x^{-\kappa-1}(u_y u_x^{-\kappa-2} - 1) - 2\,(\kappa+1)^2$

$(\kappa+2)(2\kappa+3)^{-1}u_x) + 2\,(\kappa+1)(\kappa+2)(2\kappa^2+\kappa-2)(2\kappa+3)^{-1},$

$U_3 = u_{xxy}u_{xx}^{-2}u_x^{-\kappa} - u_{xxx}u_{xx}^{-2}u_x\,(u_y u_x^{-\kappa-2} + (\kappa+1)^2) + 2\,(\kappa+2)^{-1}\,U_1$

$-(\kappa+1)(\kappa^2+\kappa+2),$

$U_4 = (\kappa+1)\,(\kappa\,U_1 - (\kappa+2)\,(U_3 - (\kappa+1)\,(u_{xxx}u_{xx}^{-2}u_x + \kappa^2 + 5\kappa + 2))),$

$U_5 = \dfrac{1}{2}\,((\kappa+2)u_{xx}^{-2}u_x^{-3\kappa-3}(u_x u_{txy} - u_{ty}) + u_{tx}u_{xx}^{-2}u_x^{-2\kappa-2}\,((\kappa+3)\,U_1 - (\kappa+2)\,(u_y u_x^{-\kappa-2}$

$+(\kappa+1)(\kappa+3))) + ((2\kappa+3)\,u_y u_x^{-\kappa-2} - 1)\,U_1^2 - (\kappa+2)((\kappa+3)\,u_y u_x^{-\kappa-2}$

$+2\kappa+1)\,U_2 - ((\kappa+1)^{-1}u_t u_x^{-2\kappa-3}(\kappa(\kappa+1)^{-1}u_y u_x^{-\kappa-2} + 2\kappa^2 + 5\kappa + 4)$

$+(\kappa+1)\,(\kappa\,u_y^2 u_x^{-2\kappa-4} - (2\kappa+3)^{-1}((2\kappa^4 + 9\kappa^3 + 7\kappa^2 - 13\kappa - 18)\,u_y u_x^{-\kappa-2}$

$-2\,(2\kappa^4 + 45\kappa^3 + 42\kappa^2 + 53\kappa + 25))))\,U_1 + (u_t u_x^{-2\kappa-3}((\kappa+1)^{-1}u_y u_x^{-\kappa-2} - 1)$

$-(\kappa+2)(u_y^2 u_x^{-2\kappa-4} - (2\kappa+3)^{-1}((2\kappa^2+5\kappa+4)u_y u_x^{-\kappa-2} - \kappa - 1)))\,U_3$

$+((\kappa+2)^{-2}\,u_t u_x^{-2\kappa-3}((\kappa+1)^{-1}u_y u_x^{-\kappa-2} + 1) + u_y^2 u_x^{-2\kappa-4}$

$-2(2\kappa^2+7\kappa+1)(2\kappa+3)\,u_y u_x^{-\kappa-2} + (2\kappa^2+8\kappa+7)(2\kappa+3)^{-1})\,U_4$

$+u_t u_x^{-2\kappa-3}((\kappa+6)\,u_y u_x^{-\kappa-2} + (\kappa+1)(4\kappa^2+9\kappa+6))$

$+(\kappa+1)^2(\kappa+2)((3\kappa+2)\,u_y u_x^{-2\kappa-4} - (2\kappa+3)^{-1}(4(\kappa^2+3\kappa+3)u_y u_x^{-\kappa-2} + 8\kappa^3$

$+36\kappa^2 + 57\kappa + 30))).$

We find contact integrable extensions [11], of the form

$$d\omega = \left(\sum_{i=0}^{3} A_i\;\theta_i + \sum{}^{*} B_{ij}\,\theta_{ij} + \sum_{s=1}^{7} C_s\;\eta_s + \sum_{j=1}^{3} D_j\;\xi^j + E\,\alpha\right)\wedge\omega$$

$$+\sum_{j=1}^{3}\left(\sum_{k=0}^{3} F_{jk}\;\theta_k + G_j\,\alpha\right)\wedge\xi^j, \qquad (5)$$

where $\sum{}^{*}$ denotes summation over all $i,j\in\mathbb{N}$ such that $1\le i\le j\le 3$ and $(i,j)\ne(3,3)$. We consider two types of such extensions. The first type consists of extensions whose coefficients in right-hard side of (5) depend on the invariants U_1, ..., U_5. The coefficients of extensions of the second type depend on U_1, ..., U_5 and on one additional function W with the differential of the form

$$dW = \sum_{i=0}^{3} H_i\;\theta_i + \sum{}^{*} I_{ij}\,\theta_{ij} + \sum_{s=1}^{7} J_s\;\eta_s + \sum_{j=1}^{3} K_j\;\xi^j + L_0\,\omega + L_1\,\alpha. \qquad (6)$$

We require Eqs. (23) and (5) or Eqs. (23), (5), and (6) to be compatible. This condition gives two contact integrable extensions of the first type

$$d\omega_1 = ((\kappa+2)^2\,(\alpha_1 - (\kappa+1)\,\theta_0) - \theta_1 - (\kappa+2)\,\theta_3) \wedge \xi^1 + \alpha_1 \wedge \xi^2$$
$$+((\kappa+2)\,(\alpha_1 - (\kappa+1)\,\theta_0) - \theta_3) \wedge \xi^3 + (\alpha_1 + \theta_2 + \theta_{22} + ((\kappa+1)(\kappa\,U_1$$
$$-(\kappa+2)\,U_3) - U_4 - (\kappa+1)(\kappa+2)(\kappa^2+6\kappa+4))(\kappa+1)^{-2}(\kappa+2)^{-2}\,\theta_0$$
$$+((\kappa+1)((\kappa+1)\,U_2 + (\kappa+2)\,U_3 - \kappa(\kappa^2+3\kappa+3)\,U_1 - \kappa(\kappa+1)(\kappa+2))$$
$$+U_4)\,(\kappa+1)^{-2}\,\xi^1 + ((\kappa+1)(\kappa^2\,U_1 + (\kappa+2)\,U_3) - \kappa\,U_4$$
$$-\kappa(\kappa+2)(3\kappa+2)(\kappa+1)^2)(\kappa+1)^{-2}(\kappa+2)^{-1}\,\xi^3) \wedge \omega_1 \tag{7}$$

and

$$d\omega_2 = ((\kappa+1)^2(\kappa+2)^2\,\theta_1 - \theta_2 + (\kappa+1)(\kappa+2)\,\theta_3) \wedge \xi^1 + \alpha_2 \wedge \xi^2$$
$$+((\kappa+1)(\kappa+2)\,\theta_2 - \theta_3) \wedge \xi^3 + (\alpha_2 + \theta_2 + \theta_{22} + ((\kappa+1)\,(\kappa\,U_1 - (\kappa+2)\,U_3)$$
$$-U_4 - (\kappa+1)(\kappa+2)(\kappa^2+6\kappa+4))(\kappa+1)^{-2}(\kappa+2)^{-2}\,\theta_0$$
$$+(U_2 - (\kappa+1)(\kappa+2)\,U_1)\,\xi^1 + (\kappa(\kappa+1)\,U_1 - U_4$$
$$-(\kappa+1)^2(\kappa+2)(3\kappa+2))(\kappa+1)^{-1}(\kappa+2)^{-2}\,\xi^3) \wedge \omega_2, \tag{8}$$

or one contact integrable extension of the second type

$$d\omega_3 = ((W+\kappa+2)^2\,\alpha_3 - (W+\kappa+2)\,(\theta_{23} + (\kappa+1)(\kappa+2)\theta_0) - \theta_1) \wedge \xi^1 + \alpha_3 \wedge \xi^2$$
$$+((W+\kappa+2)\,\alpha_3 - (\kappa+1)(\kappa+2)\,\theta_0 - \theta_3) \wedge \xi^3 + (\alpha_3 + \theta_2 + \theta_{22} + ((\kappa+1)(\kappa\,U_1$$
$$-(\kappa+2)\,U_3) - U_4 - (\kappa+1)(\kappa+2)(\kappa^2+6\kappa+4))(\kappa+1)^{-2}(\kappa+2)^{-2}\,\theta_0$$
$$-(((\kappa+1)(\kappa\,U_1 - (\kappa+2)\,U_3) - U_4 - (\kappa+1)(\kappa+2)(\kappa^2+6\kappa+4))\,W^2$$
$$+(\kappa+2)((\kappa+1)\,((\kappa-1)\,U_1 - 2\,(\kappa+2)\,(U_3 + (\kappa+1)(\kappa^2+5\kappa+3)) - 2\,U_4))\,W$$
$$+(\kappa+2)^2((\kappa+1)(\kappa(\kappa^2+3\kappa+3)\,U_1 - (\kappa+1)\,U_2 - (\kappa+2)\,(U - \kappa^2(\kappa+1)))$$
$$-U_4)(\kappa+1)^{-2}(\kappa+2)^{-2}\,\xi^1 - ((\kappa+1)(\kappa\,U_1 - (\kappa+2)\,(U_3 + (\kappa+1)(\kappa^2+6\kappa+4)))$$
$$-U_4)(\kappa+1)^{-2}(\kappa+2)^{-2}\,\xi^3) \wedge \omega_3, \tag{9}$$

$$dW = -(\kappa+1)\,W\,(\alpha_3 + \theta_0 + \theta_2) + Z\,\xi^2 + (W+\kappa+2)(Z + (\kappa+1)\,W)\,\xi^3$$
$$+(W+\kappa+2)((W+\kappa+2)\,Z + (\kappa+1)\,W\,(W - (\kappa+2)^{-1}\,U_1 + 3\kappa+4))\,\xi^1$$
$$+(Z - (\kappa\,U_1 - (\kappa+2)\,(U_3 + (\kappa+1)^2(\kappa+6))$$
$$-(\kappa+1)^{-1}U_4)(\kappa+2)^{-1}\,W)\,\omega_3 \tag{10}$$

with a parameter Z.

The inverse third fundamental Lie theorem in Cartan's form [16, §26; 15, p. 394], ensures existence of forms ω_1, ω_2, ω_3 satisfying Eqs. (7), (8), and (9). Since the forms (4) are known explicitly, it is not hard to find the forms ω_i. We have the following solutions to Eqs. (7), (8), and (9), respectively:

$$\omega_1 = \frac{u_{xx}}{u_x q_x}\Bigg(dq - \left(\frac{u_t}{u_x} + (\kappa+2)\left(u_y u_x^{\kappa} + \frac{\kappa+1}{2\kappa+3}\, u_x^{2\kappa+2}\right)\right) q_x\, dt - q_x\, dx$$

$$-\left(\frac{u_y}{u_x} + u_x^{\kappa+1}\right) q_x\, dy\Bigg), \tag{11}$$

$$\omega_2 = \frac{u_{xx}}{u_x r_x}\Bigg(dr - \left(\frac{u_t}{u_x} - (\kappa+1)(\kappa+2)\left(u_y u_x^{\kappa} - \frac{1}{2\kappa+3}\, u_x^{2\kappa+2}\right)\right) r_x\, dt - r_x\, dx$$

$$-\left(\frac{u_y}{u_x} - (\kappa+1)\, u_x^{\kappa+1}\right) r_x\, dy\Bigg), \tag{12}$$

and

$$\omega_3 = \frac{u_{xx}}{u_x s_x}\Bigg(ds - \Bigg(\frac{(\kappa+2)^2}{2\kappa+3}\, s_x^{2\kappa+3} - (\kappa+2)\left(\frac{u_y}{u_x} + u_x^{\kappa+1}\right) s_x^{\kappa+2}$$

$$+\left(\frac{u_t}{u_x} + (\kappa+2)\, u_x^{\kappa} u_y + \frac{(\kappa+1)(\kappa+2)}{2\kappa+3} u_x^{2\kappa+2}\right) s_x\Bigg)\, dt - s_x\, dx$$

$$+\left(s_x^{\kappa+2} - \left(\frac{u_y}{u_x} + u_x^{\kappa+1}\right) s_x\right) dy\Bigg) \tag{13}$$

with $W = s_x^{\kappa+1} u_x^{-\kappa-1}$.

The forms (11), (12), (13) are equal to zero if and only if the following overdetermined systems of PDEs are satisfied:

$$\begin{cases} q_t = \left(\dfrac{u_t}{u_x} + (\kappa+2)\left(u_y u_x^{\kappa} + \dfrac{\kappa+1}{2\kappa+3}\, u_x^{2\kappa+2}\right)\right) q_x, \\ q_y = \left(\dfrac{u_y}{u_x} + u_x^{\kappa+1}\right) q_x, \end{cases} \tag{14}$$

$$\begin{cases} r_t = \left(\dfrac{u_t}{u_x} - (\kappa+1)(\kappa+2)\left(u_y u_x^{\kappa} - \dfrac{1}{2\kappa+3}\, u_x^{2\kappa+2}\right)\right) r_x, \\ r_y = \left(\dfrac{u_y}{u_x} - (\kappa+1)\, u_x^{\kappa+1}\right) r_x, \end{cases} \tag{15}$$

$$\begin{cases} s_t = \dfrac{(\kappa+2)^2}{2\kappa+3}\, s_x^{2\kappa+3} - (\kappa+2)\left(\dfrac{u_y}{u_x} + u_x^{\kappa+1}\right) s_x^{\kappa+2} \\ +\left(\dfrac{u_t}{u_x} + (\kappa+2)\, u_x^{\kappa} u_y + \dfrac{(\kappa+1)(\kappa+2)}{2\kappa+3} u_x^{2\kappa+2}\right) s_x, \\ s_y = -s_x^{\kappa+2} + \left(\dfrac{u_y}{u_x} + u_x^{\kappa+1}\right) s_x. \end{cases} \tag{16}$$

These systems are compatible whenever u is a solution to Eq. (1), so these systems define differential coverings over (1).

Expressing u_t and u_y from (14) and cross-differentiating yields

$$q_{yy} = q_{tx} + \left((\kappa+1)\,\frac{q_y^2}{q_x^2} - \frac{q_t}{q_x}\right) q_{xx} - \kappa\,\frac{q_y}{q_x}\,q_{xy}. \tag{17}$$

Previously Eq. (17) and the Bäcklund transformation (14) were found in [12] by means of another method.

From Eqs. (15) we have

$$\begin{cases} u_t = \left(\dfrac{r_t}{r_x} + (\kappa+1)(\kappa+2)\left(\dfrac{r_y}{r_x}\,u_x^{\kappa+1} + \dfrac{(\kappa+2)(2\kappa+1)}{2\kappa+3}\,u_x^{2\kappa+2}\right)\right) u_x, \\ u_y = \left(\dfrac{r_y}{r_x} + (\kappa+1)\,u_x^{\kappa+1}\right) u_x. \end{cases} \tag{18}$$

The compatibility condition for this system is

$$(u_t)_y - (u_y)_t = -(\kappa+1)(\kappa+2)\,u_x^{\kappa+2} r_x^{-2}\left(G\,r_x - \kappa\,(\kappa+2)\,u_x^{\kappa+1}\,(r_y\,r_{xx} - r_x\,r_{xy})\right) = 0, \tag{19}$$

where

$$G = r_{yy} - r_{tx} - \left((\kappa+1)\,\frac{r_y^2}{r_x^2} - \frac{r_t}{r_x}\right) r_{xx} + \kappa\,\frac{r_y}{r_x}\,r_{xy}.$$

When $\kappa = 0$, system (18) is compatible whenever $G = 0$, that is, whenever r is a solution to Eq. (17). When $\kappa \neq 0$, Eq. (19) entails $u_x^{\kappa+1} = H$ with

$$H = -\kappa^{-1}(\kappa+2)^{-2}\,G\,r_x\,(r_y\,r_{xx} - r_x\,r_{xy})^{-1}.$$

Substituting this into (18) gives a system of PDEs with the compatibility condition

$$\begin{aligned} &\kappa\,(2\kappa+3)\,r_x^2\,H_t - \kappa\,(\kappa+2)\,r_x\left(2(\kappa+2)(2\kappa+1)\,r_x H + (2\kappa+3)\,r_y\right) H_y \\ &+\kappa\left((\kappa+1)(\kappa+2)^2(2\kappa+1)\,r_x^2\,H^2 + 2(\kappa+2)(2\kappa+1)\,r_x r_y H\right. \\ &\left.-(2\kappa+3)(r_t r_x - (\kappa+2)\,r_y^2)\right) H_x - (\kappa+1)\,((2\kappa^2+5\kappa+1)\,r_x G \\ &+\kappa(2\kappa+3)(r_x r_{tx} - r_t r_{xx}))\,H - (2\kappa+3)\,r_y\,G = 0. \end{aligned} \tag{20}$$

Thus Eqs. (15) define a Bäcklund transformation from Eq. (1) to the third order equation (20) for r.

Finally, excluding u from (16) shows that s is a solution to the same equation (1). So, (16) defines a Bäcklund auto-transformation for Eq. (1).

The Bäcklund auto-transformation (16) allows one to find a recursion operator for symmetries of Eq. (1), see [8]. Charactersitics of (local contact) symmetries are solutions to the linearization

$$\bar{\mathbb{D}}_y^2(\varphi) = \bar{\mathbb{D}}_t\bar{\mathbb{D}}_x(\varphi) + \left(\frac{(\kappa+1)\,u_y^2}{u_x^2} - \frac{u_t}{u_x} + \kappa\,u_x^{\kappa}u_y + \frac{(\kappa+1)^2}{2\,\kappa+3}\,u_x^{2(\kappa+1)}\right)\bar{\mathbb{D}}_x^2(\varphi) - \frac{u_{xx}}{u_x}\,\bar{\mathbb{D}}_t(\varphi)$$

$$-\kappa\left(\frac{u_y}{u_x}+u_x^{\kappa+1}\right)\bar{\mathbb{D}}_x\bar{\mathbb{D}}_y(\varphi)+\left(2\,(\kappa+1)\,\frac{u_y}{u_x^2}+\kappa\,u_x^{\kappa}\right)\bar{\mathbb{D}}_y(\varphi)$$
$$+\left(\left(\frac{u_t}{u_x^2}-2\,(\kappa+1)\,\frac{u_y^2}{u_x^3}+\kappa u_x^{\kappa-1}u_y+\frac{2\,(\kappa+1)^3}{2\kappa+3}\,u_x^{2\kappa+1}\right)u_{xx}\right.$$
$$\left.+\kappa\left((\kappa+1)\,u_x^{\kappa}-\frac{u_y}{u_x^2}\right)u_{xy}\right)\bar{\mathbb{D}}_x(\varphi) \tag{21}$$

of Eq. (1). Then the linearization

$$\bar{\mathbb{D}}_t(\psi)=(\kappa+2)\left(u_x^{\kappa}s_x+\frac{s_x^{\kappa+2}}{u_x}\right)\bar{\mathbb{D}}_y(\varphi)+\left((\kappa+2)^2s_x^{2(\kappa+1)}-(\kappa+2)^2\left(\frac{u_y}{u_x}+u_x^{\kappa+1}\right)s_x^{\kappa+1}\right.$$
$$\left.+\frac{u_t}{u_x}+(\kappa+2)\left(u_x^{\kappa}\,u_y+\frac{\kappa+1}{2\kappa+3}u_x^{2(\kappa+1)}\right)\right)\bar{\mathbb{D}}_x(\psi)+\frac{s_x}{u_x}\,\bar{\mathbb{D}}_t(\varphi)-\frac{u_t}{u_x^2}\,s_x\,\bar{\mathbb{D}}_x(\varphi)$$
$$+(\kappa+2)\left(\left(\frac{u_y}{u_x^2}-(\kappa+1)\,u_x^{\kappa}\right)s_x^{\kappa+1}+\left(\kappa\,u_x^{\kappa-1}u_y+\frac{2\,(\kappa+1)^2}{2\,\kappa+3}\,u_x^{2\kappa+1}\right)\right)s_x\,\bar{\mathbb{D}}_x(\varphi),$$
$$\bar{\mathbb{D}}_y(\psi)=\left(\frac{u_y}{u_x}-(\kappa+2)\,s_x^{\kappa+1}+u_x^{\kappa+1}\right)\bar{\mathbb{D}}_x(\psi)+\frac{s_x}{u_x}\,\bar{\mathbb{D}}_y(\varphi)-\left(\frac{u_y}{u_x^2}-(\kappa+1)\,u_x^{\kappa}\right)s_x\,\bar{\mathbb{D}}_x(\varphi) \tag{22}$$

of system (16) maps solutions φ of (21) to its solutions ψ, and vice versa. Thus (22) is a recursion operator for symmetries of (1). It is easy to verify by direct computation, that each local contact symmetry of Eq. (1) has a lift to a symmetry of system (16). So the covering defined by system (16) does not contain non-removable parameters [8]. Therefore Eq. (1) provides an example of PDE that possesses a recursion operator which corresponds to a covering without non-removable parameters.

APPENDIX

The structure equations of the symmetry pseudo-group of Eq. (1) read

$$d\theta_0=(\theta_{22}+(U_2-(\kappa+1)^2\,(U_1-2\,(\kappa+1))\,\xi^1-(U_4+(\kappa+1)(\kappa+2)\,U_3-\kappa\,(\kappa+1)\,U_1$$
$$+(\kappa+1)^2(\kappa+2)\,(2\kappa+3))\,(\kappa+1)^{-1}\,(\kappa+2)^{-1}\,\xi^3)\wedge\theta_0+\xi^1\wedge\theta_1+\xi^2\wedge\theta_2+(\kappa+1)^2(\kappa+2)$$
$$(\kappa^2+6\,\kappa+4))\,(\kappa+1)^{-2}(\kappa+2)^2\,\xi^2-(U_4-\kappa\,(\kappa+1)\,U_1+\xi^3\wedge\theta_3,$$
$$d\theta_1=(\kappa+1)\,(2\,\theta_1+(\kappa+1)(\kappa+2)^2\,\theta_2-(\kappa+2)\,\theta_3))\wedge\theta_0+\xi^1\wedge\theta_{11}+\xi^3\wedge\theta_{13}$$
$$+(\kappa+1)(\kappa+2)\,\theta_2\wedge\theta_3-(2\,(\kappa+1)\,(\theta_2-2\,(U_1-2\,(\kappa+1)(\kappa+2))\,\xi^1+\xi^2)$$
$$+((2\,\kappa+3)\,U_1-(\kappa+1)(\kappa+2)(3\,\kappa+4))(\kappa+2)^{-1}\,\xi^3)\wedge\theta_1+\xi^2\wedge(U_3\,\theta_0+U_1\,\theta_3+\theta_{12}),$$
$$d\theta_2=\theta_0\wedge\theta_{22}+\left((\kappa+1)\,\big(U_1-2\,(\kappa+1)(\kappa+2)\big)\,\xi^1+\frac{1}{2}\,U_1\,\xi^3\right)\wedge\theta_2$$
$$+\xi^1\wedge\theta_{12}+\xi^2\wedge\theta_{22}+\xi^3\wedge\theta_{23},$$
$$d\theta_3=((\kappa+1)\,(\theta_3-(\kappa+1)(\kappa+2)\,\theta_2+(\kappa+1)(\kappa+2)((\kappa+3)\,U_1-(\kappa+2)\,(U_2+2))\,\xi^1)$$
$$+U_3\,\xi^2+(U_2+U_4)\,\xi^3-(\kappa+1)(\kappa+2)\,\theta_{22})\wedge\theta_0+\left((\kappa+1)\,\theta_3+\frac{1}{2}\,U_1\,\xi^2\right)\wedge\theta_2$$
$$+(\kappa+1)(2\,(U_1-2\,(\kappa+1)(\kappa+2))\,(\xi^1+(\kappa+2)^{-1}\,\xi^3)-\xi^2)\wedge\theta_3+\xi^1\wedge\theta_{13}$$
$$+\xi^2\wedge\theta_{23}+\xi^3\wedge\theta_{12},$$

$$d\theta_{11} = \eta_1 \wedge \xi^2 + \eta_2\xi^3 + \eta_3 \wedge \xi^1 + ((4\,U_4 - (\kappa+1)(\kappa-2)\,U_1 - \kappa(\kappa+1)^2(\kappa^2-4)$$
$$+(2\kappa+1)\,U_2)\,\theta_1 - (\kappa+1)^2(\kappa+2)\,(U_1 - 2\,(\kappa+1)(\kappa+2))\,(\theta_2 + (\kappa+1)(\kappa+2)\,\theta_3)$$
$$+(\kappa^2-1)(\kappa+2)\,\theta_{13} + (\kappa+1)(\kappa\,(2\,U_5 + 3\,(\kappa+2)\,U_2) - U_1U_2$$
$$-(\kappa+1)^2(\kappa+2)(3\,\kappa-2)((\kappa+3)\;U_1 - 2\,(\kappa+1)(\kappa+2)))\,\xi^2 + ((\kappa+1)(\kappa\,U_1$$
$$-(\kappa+2)\,U_3 + (\kappa+1)(\kappa+2)(3\kappa^2+6\kappa+4)) - U_4)(\kappa+1)^{-2}(\kappa+2)^{-2}\,\theta_{11}) \wedge \theta_0$$
$$+((\kappa+1)(4\,U_1 + (\kappa+1)(\kappa+2)\,(11\kappa+14))\,\theta_2 - (\kappa+1)(2\,U_1 - (\kappa+1)(\kappa+2))\,\theta_3$$
$$-(2\kappa+1)\,\theta_{12} + 4(\kappa+1)(\kappa+2)\,\theta_{23} - (4\,U_4 - (\kappa+1)(2\,\kappa^2+3\,\kappa-4)\,U_1$$
$$+(2\kappa+1)\,U_2 + 4\,\kappa(\kappa+1)^2(\kappa+2) - (2\kappa+3)(\kappa+2)^{-1}\,U_1^2)\,\xi^2$$
$$-(2(2\kappa+3)(\kappa+2)^{-1}\,U_5 + (3\kappa+2)(\kappa+1)\,(U_2 - (\kappa+3)(\kappa+1)\,U_1$$
$$+2(\kappa+2)(\kappa+1)^2)\,\xi^3) \wedge \theta_1 + ((\kappa+2)(\kappa+1)^2\,(U_1 - 2\,(\kappa+1)\,(\kappa+2))\,\theta_3$$
$$+(4\,\kappa+5)\,\theta_{11} - (\kappa+1)(\kappa+2)\,\theta_{13}) \wedge \theta_2 + ((\kappa+2)\,\theta_{13} - (2\,U_5$$
$$+(\kappa+1)(U_1^2 - 2(\kappa+2)(U_2 + (\kappa+1)(\kappa+4)\,U_1 + 2(\kappa+1)^2(\kappa+2))))\,\xi^2) \wedge \theta_3$$
$$-(\theta_{22} - ((\kappa+1)(\kappa+9)\,U_1 - U_2 - 14(\kappa+2)\,(\kappa+1)^2)\,\xi^1 - ((\kappa+1)(\kappa\,U_1$$
$$-(\kappa+2)\,(U_3 - (3\kappa^2+6\kappa+4)(\kappa+1))) - U_4)(\kappa+1)^{-2}(\kappa+2)^{-2}\,\xi^2$$
$$+(3\,(U_1 - (\kappa+1)(\kappa+2)) + (\kappa+1)^{-1}(\kappa+2)^{-1}\,U_4)\,\xi^3) \wedge \theta_{11}$$
$$+((\kappa+1)(U_1 - 2\,(\kappa+1)(\kappa+2))\,\theta_{12} - 2\,U_1\,\theta_{13}) \wedge \xi^2$$
$$d\theta_{12} = \eta_1 \wedge \xi^1 + \eta_4 \wedge (\theta_0 + \xi^2) + \eta_7 \wedge \xi^3 + (\,(U_1 - (\kappa+4)(\kappa+2)(\kappa+1))\,\theta_{22}$$
$$+(\kappa+1)((\kappa+2)\,\theta_{23}) + (U_4 + \frac{1}{2}\,(\kappa+1)(\kappa^2+2\kappa+2)\,U_1 + \kappa(\kappa+1)^2(\kappa+2))\,\theta_2$$
$$+((\kappa+1)\,(\kappa\,U_1 - (\kappa+2)\,(U_3 - \kappa^2(\kappa+1))) - U_4)(\kappa+1)^{-2}(\kappa+2)^{-2}\,\theta_{12}) \wedge \theta_1$$
$$+((2\,\kappa+3)\,\theta_{12} - (\kappa+1)(\kappa+2)\,\theta_{23} - \frac{1}{2}\,U_1\,(\kappa+2)\,\theta_3 - U_5\,\xi^3 + (\frac{1}{2}\,U_1^2 - (\kappa+1)\,U_1$$
$$-U_4 - \kappa(\kappa+2)(\kappa+1)^2)\,\xi^2) \wedge \theta_2 + (\kappa+2)\,(\theta_{23} - (\kappa+1)\,\theta_{22}) \wedge \theta_3$$
$$-(\theta_{22} - ((\kappa+1)(\kappa+5)\,U_1 - U_2 - 6(\kappa+2)(\kappa+1)^2)\,\xi^1 + ((\kappa+1)(\kappa\,U_1$$
$$-(\kappa+2)(U_3 - \kappa^2(\kappa+1)) - U_4)(\kappa+2)^{-2}(\kappa+1)^{-2}\,\xi^2 - \frac{1}{2}\,((3\kappa+8)(\kappa+1)\,U_1$$
$$+2\,U_4 - 4(\kappa+2)(\kappa+1)^2)(\kappa+1)^{-1}(\kappa+2)^{-1}\,\xi^3) \wedge \theta_{12}$$
$$+2(\kappa+1)(U_1 - 2\,(\kappa+2)(\kappa+1))\,(\theta_{22} \wedge \xi^2 + \theta_{23} \wedge \xi^3) - U_1\,\theta_{23} \wedge \xi^2$$
$$d\theta_{13} = \eta_1 \wedge \xi^3 + \eta_2 \wedge \xi^1 + \eta_7 \wedge \xi^2 + ((\kappa+1)^2(\kappa+2)^2\,\theta_{23} - U_3\,\theta_1 - (\kappa+1)(\kappa+2)\,\theta_{12}$$
$$-\frac{1}{2}\,(\kappa+1)(\kappa+2)((\kappa+4)(\kappa+1)\,U_1 - 4\,(U_2 + (\kappa+2)(\kappa+1)))\,\theta_2$$
$$-((\kappa^2-1)\,U_1 + U_2 - U_4 - 2(\kappa+2)(2\kappa+1)(\kappa+1)^2)\,\theta_3 + ((\kappa+1)\,(\kappa\,U_1$$
$$+(\theta_{23} - (\kappa+1)(\kappa+2)\,\theta_{22} + (U_2 + U_4)\,\xi^3 + \frac{1}{2}\,(U_1 - 2(\kappa+2)(\kappa+1)^2)\,\theta_2$$
$$+U_3\,\xi^2) \wedge \theta_1 + ((3\kappa+4)\,\theta_{13} - (\kappa+1)(\kappa+2)\,\theta_{12} - U_5\,\xi^2 - \frac{1}{2}\,(\kappa+1)((3\kappa+4)\,U_1$$
$$-4\,(\kappa+1)(\kappa+2))\,\theta_3) \wedge \theta_2 + (\theta_{12} + (\kappa+1)(\kappa+2)\,\theta_{23} + ((\kappa+1)(2\,U_1^2$$
$$+(\kappa+2)((\kappa^2-\kappa-4)\,U_1 - U_2 - 2\kappa(\kappa+1)(\kappa+2))) - 2(\kappa+2)U_4)(\kappa+2)^{-1}\,\xi^2$$
$$+((\kappa+1)(\kappa+2)(5\kappa+2)((\kappa+1)(\kappa+3)\,U_1 - U_2 - 2(\kappa+2)(\kappa+1)^2)$$
$$-4(\kappa+1)U_5)(\kappa+2)^{-1}\,\xi^3) \wedge \theta_3 + \frac{3}{2}\,U_1\,\xi^2 \wedge \theta_{12} - (\theta_{22} - ((\kappa+1)((\kappa+7)\,U_1$$
$$-10(\kappa+2)(\kappa+1)) - U_2)\,\xi^1 + ((\kappa+1)(\kappa U_1 - (\kappa+2)(U_3 - (2\kappa^2+3\kappa+2)(\kappa+1))$$

$$+U_4)(\kappa+1)^{-1}(\kappa+2)^{-1}\,\xi^3)\wedge\theta_{13}+(\kappa+1)(U_1-2(\kappa+1)(\kappa+2))\,\theta_{23}\wedge\xi^2$$
$$-(\kappa+2)\,(U_3-(2\kappa^2+3\kappa+2)(\kappa+1))-U_4)(\kappa+1)^{-2}(\kappa+2)^{-2}\,\theta_{13})\wedge\theta_0$$
$$-U_4)(\kappa+1)^{-2}(\kappa+2)^{-2}\,\xi^2-((3\kappa+5)(\kappa+1)U_1-2(\kappa+2)(2\kappa+3)(\kappa+1)^2$$
$$d\theta_{22}=\eta_4\wedge\xi^1+\eta_5\wedge(\theta_0+\xi^2)+\eta_6\wedge\xi^3-(U_4-(\kappa+1)(\kappa\,U_1+(\kappa+2)\,U_3$$
$$+(\kappa+1)(\kappa+2)(\kappa^2+6\kappa+4)))(\kappa+1)^{-2}(\kappa+2)^{-2}\,\theta_{22}\wedge(\theta_0+\xi^2)$$
$$+(((\kappa+1)^2(U_1+2\kappa+4))-U_2)\,\xi^1-((\kappa+1)(\kappa\,U_1+(\kappa+1)(\kappa+2)(3\kappa+2))$$
$$-U_4)(\kappa+1)^{-1}(\kappa+2)^{-1}\,\xi^3)\wedge\theta_{22}$$
$$d\theta_{23}=\eta_4\wedge\xi^3+\eta_6\wedge(\theta_0+\xi^2)+\eta_7\wedge\xi^1+\frac{1}{2}\,(U_1\,\theta_{22}+(\kappa\,U_4-(\kappa+1)(\kappa^2\,U_1$$
$$-2(\kappa+2)\,U_3+\kappa(\kappa+2)(3\kappa+2)(\kappa+1)^2)(\kappa+1)^{-1}(\kappa+2)^{-2}\,\theta_2-2\,(\kappa(\kappa+1)U_1^2$$
$$+((\kappa+2)\,U_3+U_4+(\kappa+2)(\kappa^2-3\kappa-2)(\kappa+1)^2)\,U_1+\kappa(\kappa+1)(\kappa+2)(U_4$$
$$+(\kappa+2)\,U_3))(\kappa+2)^{-2}\,\xi^3-2\,(U_4-(\kappa+1)(\kappa\,U_1+(\kappa+2)\,U_3$$
$$+(\kappa+1)(\kappa+2)(3\kappa+2)))(\kappa+1)^{-2}(\kappa+2)^{-2}\,\theta_{23})\wedge\theta_0+\frac{1}{4}\,(6\,(\kappa+2)\,\theta_{23}$$
$$-8\,(\kappa+1)(\kappa+2)\,\theta_{22}-2\,(\kappa\,U_4-(\kappa+1)(\kappa^2\,U_1-2(\kappa+2)\,U_3$$
$$+\kappa(\kappa+1)(\kappa+2)(3\kappa+2))((\kappa+1)^{-1}(\kappa+2)^{-1}\,\xi^2+(\kappa\,U_1^2-(\kappa+2)\,(2(\kappa+1)(\kappa^2$$
$$+4\kappa+2)\,U_1+2(\kappa+4)\,U_2+4\,U_4))(\kappa+2)^{-1}\,\xi^3)\wedge\theta_2+(\kappa(\kappa+1)U_1+(\kappa+2)U_3$$
$$-U_4-(\kappa+2)(3\kappa+2)(\kappa+1)^2)(\kappa+2)^{-1}\,\theta_3\wedge\xi^3+(\theta_{23}-\frac{1}{2}\,U_1\,\xi^2-2(\kappa+1)(U_1$$
$$-2(\kappa+1)(\kappa+2))\,\xi^3)\wedge\theta_{22}+(((\kappa+3)(\kappa+1)U_1-U_2-2(\kappa+2)(\kappa+1)^2)\,\xi^1$$
$$+(U_4-(\kappa+1)(\kappa\,U_1+(\kappa+2)\,U_3+(\kappa+2)(3\kappa+2)(\kappa+1))(\kappa+1)^{-2}(\kappa+2)^{-2}\,\xi^2$$
$$+\frac{1}{2}\,(3(\kappa+1)(\kappa+2)U_1+2U_4-2(\kappa+1)^2(\kappa+2)^2)(\kappa+1)^{-1}(\kappa+2)^{-1}\,\xi^3)\wedge\theta_{23}$$
$$d\xi^1=(\theta_{22}+(2\,\kappa+3)\,\theta_2-((\kappa+1)((\kappa+3)\,U_1-2\,(\kappa+2))+U_4)\,((\kappa+1)(\kappa+2))^{-1}\xi^3$$
$$+((\kappa+1)\,(\kappa\,U_1-(\kappa+2)\,(U_3-\kappa^2(\kappa+1)))-U_4)\,((\kappa+1)(\kappa+2))^{-2}\,(\theta_0+\xi^2))\wedge\xi^1,$$
$$d\xi^2=(\theta_1-(\kappa+1)(\kappa+2)\,\theta_3-(\kappa+1)^2(\kappa+2)^2\,\theta_0)\wedge\xi^1+(\theta_2+\theta_{22}+(U_4-\kappa(\kappa+1)\,U_1$$
$$-(\kappa+1)^2(\kappa+2)(3\kappa+2))(\kappa+1)^{-1}(\kappa+2)^{-1}\,\xi^3-(U_4+(\kappa+1)(\kappa+2)\,U_3$$
$$-\kappa(\kappa+1)\,U_1(\kappa+1)^2(\kappa+2)(\kappa^2+6\,\kappa+4))(\kappa+1)^{-2}(\kappa+2)^{-2}\,\theta_0$$
$$+(U_2-(\kappa+1)(\kappa+2)\,U_1)\,\xi^1)\wedge\xi^2+(\theta_3-(\kappa+1)(\kappa+2)\,\theta_3)\wedge\xi^3$$
$$d\xi^3=((\kappa+2)\,((\kappa+1)(\kappa\,\theta_0-2\,\theta_2)+\theta_3)+(\kappa(\kappa+4)\,U_1-U_2-2\,(\kappa+1)^2(\kappa+2))\,\xi^3$$
$$-U_1\,\xi^2)\wedge\xi^1+(\theta_{22}+(\kappa+2)\,\theta_2-(U_4+(\kappa+1)(\kappa+2)\,U_3-\kappa(\kappa+1)\,U_1$$
$$+(\kappa+1)^2(\kappa+2)(3\kappa+2))(\kappa+1)^{-2}(\kappa+2)^{-2})\,(\theta_0+\xi^2))\wedge\xi^3 \qquad (23)$$

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КОНТАКТНЫЕ ИНТЕГРИРУЕМЫЕ РАСШИРЕНИЯ ГРУППЫ ПСЕВДОСИММЕТРИЙ И НАКРЫТИЯ R-го ДВАЖДЫ МОДИФИЦИРОВАННОГО БЕЗДИСПЕРСИОННОГОУРАВНЕНИЯ КАДОМЦЕВА – ПЕТВИАШВИЛИ

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Найдены контактные интегрируемые расширения группы псевдосимметрий и накрытия r-того дважды модифицированного бездисперсионного уравнения Кадомцева – Петвиашвили. Одно из накрытий приводит к преобразованию Бэклунда и оператору рекурсии для исследуемых уравнений.

Ключевые слова: псевдогруппа Ли, дифференциальное накрытие, контактное интегрируемое расширение, преобразование Бэклунда.

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