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THE LINEAR MODEL USING IN CRACK GROWTH SIMULATION UNDER VARIABLE-AMPLITUDE LOADING[1]

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The paper shows the possibility of using Paris-Erdogan equation for simulation fatigue crack growth under random loading. The considered equation has introduced the effective stress intensity factor range. The introduced methodology for crack growth simulation is based on the concept of a "basic" random loading. The discussed methodology included consideration of overloads influence on the fatigue crack growth. The theoretical model is based on the experimental researches of fatigue crack growth under random loading that have been realized during specimens fatigue tests of two Al-based alloys (D16chAT - the same as 2024-T3, and B95ATB – 7075-T6). In the tests the specimens of a center cracked panel were used. In all cases the random loading has been considered as Gaussian processes of cyclic loading with introduced and discussed parameters of investigated processes. As a result of investigations the model for estimation crack growth period in the different random of irregular cyclic loads was introduced.

Keywords: crack growth life, variable-amplitude loading, fatigue strength, aluminum alloy.

Introduction

Theoretical-experimental researches of crack growth duration usually include two stages. The first stage is changing of real loads spectrum for the schematized one by different models. The aim of such models is to decrease testing time for structures. Spectra of random loads which represent several models used for test of aircraft structures, are shown in Fig. 1.

Harmonic loading (or constant amplitude of loads) and typical block loads (Fig. 1a) are used for comparing test results of aircraft structures of different design. The "typical flight", shown in Fig.1b, mainly used for lower wing sheet tests and calculations and, a block of loads "TWIST"-type (Fig. 1f) can also be considered. Typical blocks of flight-type cyclic loads for wing lower sheets in wing-root-location used in tests for two different aircraft area are shown in Fig. 2.

Fig. 1e shows a wing lower panel tensometric stress record at "bumpy flight". It is clear that spectrum of operated loads has a principle difference with their modeled programs. That is why the first level of mistakes in crack grows duration estimation related to changing real operated loads spectra of their modeling.

The second stage, mathematical models construction for crack growth duration estimation, which, took into account the design philosophy of observed structures, loading conditions and in-flight operations. In the constructed mathematical models empirical parameters were used which should be experimentally estimated. That's why the next possible mistakes in theoretical-experimental estimations of crack growth duration are inaccuracy in estimations of parameters of cyclic loading processes.

This article analyzed existent cracks growth estimation models under overloads, introduced a new approach to crack growth modeling. The article shows good correlation of calculated results by the introduced model in comparison with experiment results performed under random loading. The principle of linear damages accumulation summering possibility to use for crack growth duration estimation is discussed for cases of "typical flight" program and stationary Gauss processes.

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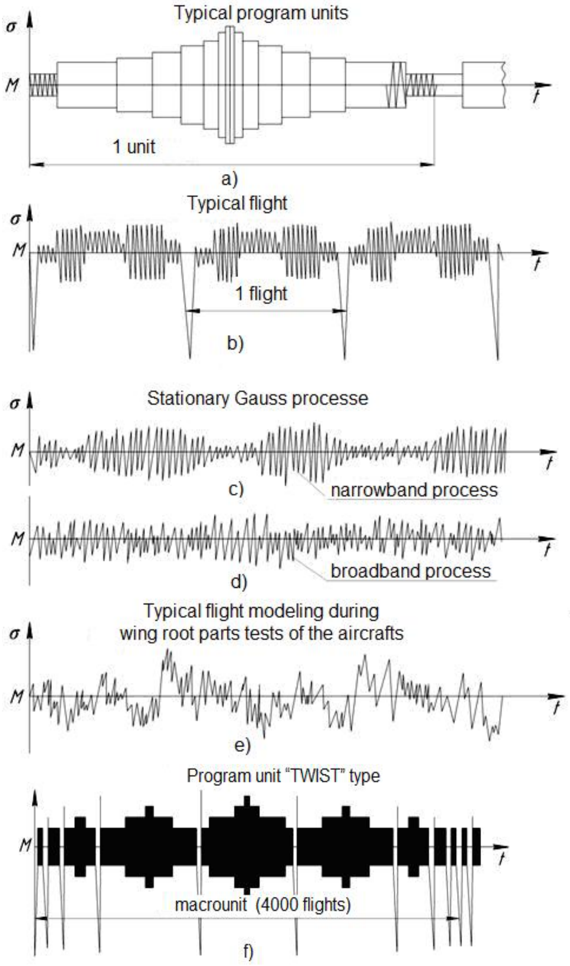
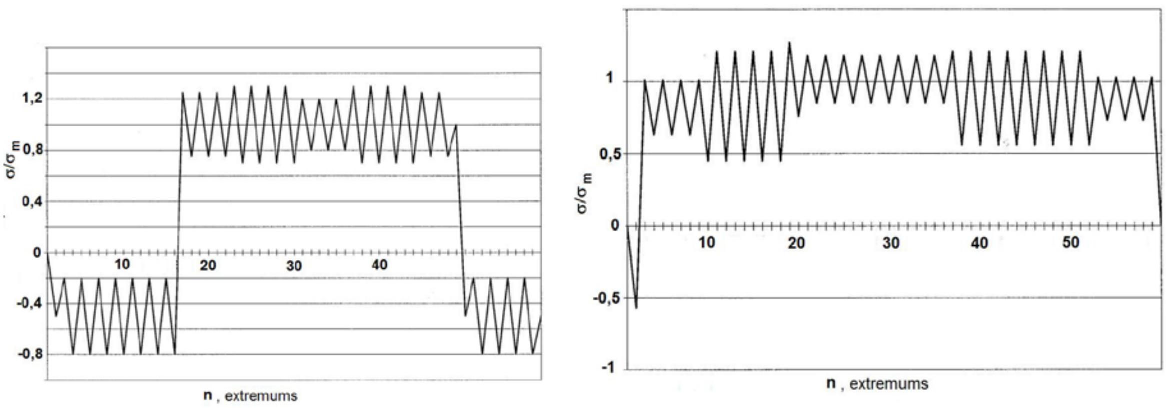


Fig. 1. Operational loads spectrum models



a b

Fig. 2. Wing root zones of the lower panels operational stress imitating system

Crack growth duration estimation logic circuit

The procedure of the fatigue life modeling is shown on the Fig. 3. It consists of four parts [2]. Each step can provoke the appearance of errors in the calculation of the crack growth period.

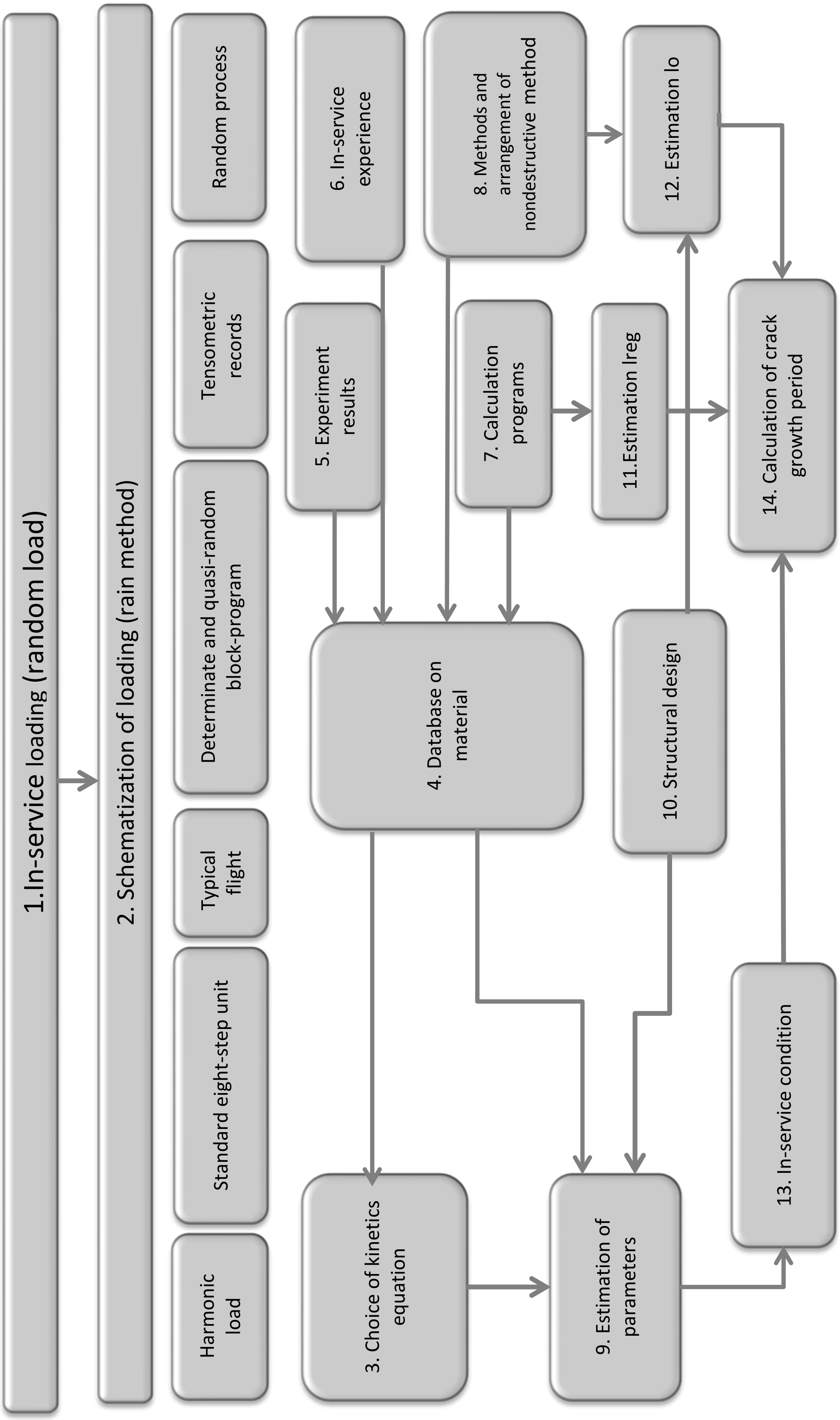


Fig. 3. The block diagram of the modeling of crack growth duration

The **first** step is the replacement the real spectrum loading by one of the simplified models (units 1, 2). The following models can be used: typical standard blocks of loading using in practice; typical flying; random loading with different structure; strain gauge recorders; simulation program as TWIST (Fig. 1, 2) [4; 5; 9]. The using of these simplified models can give the errors.

The **second** step includes the development of the mathematic model of crack growth simulation (units 3, 9) There are more fifty models quoted in the literature. Model for crack growth simulation is chosen in dependents on in-service acting cyclic loads. It can be equations, which can consider simply a case of cyclic loading without cycles loads interaction effects (equation types of Paris, Foreman et al), or models, which take into account these effects (models of Willer, Willenborg, Matsuoka, etc). In this case appear errors connected with the incorrect choose of the model type, and with the inaccurate receiving the models parameters "C" and "n" estimation. It is necessary to study the structural design of aviation controls (unit 10) and to choose the useful model that will can to give own errors. The experiments results (unit 5) and the in-service experience (unit 6) will should to be considerate. Usually the simplified model of this is using also gives errors.

The **third** step of the modeling is concerned using simulation programs and data bases for consideration of the features of the specific models (units 4, 7).

The aim of crack growth modeling is to receive the duration of crack growth from initial size to critical length. Thus the estimation of critical size and detectable of length crack with using the methods and arrangements of nondestructive control. The estimation errors of critical and detectable length crack can will be too.

The analyses of crack growth modeling errors are given in [10].

In general case, crack growth equation can be written as

$$\frac{dl}{dN} = f(K, \bar{p}, \bar{q}). \quad (1)$$

In equation (1), l - crack length; N– loading cycles quantity; $\bar{p} = (c, n, K_*, K_{th}, K_{fc} \dots)$ -vector of crack cycle closing ability parameters; c, n – experimentally determined parameters of kinetic equation; K_* - critical value of stress intensity factor (SIF); K_{th} – threshold of SIF; K_{fc} – maximum value of SIF for regular crack growth; $\bar{q} = (E, \sigma_{02}, \sigma_{в} \dots)$ -vector, which defines material mechanical properties; E– Modulus elasticity; σ_{02}– yield strength; $\sigma_{в}$ – ultimate tensile stress; K - value has meaning of SIF range, or SIF maximal value and is determined by relation

$$K = K_0 \varphi_1(N, l) \varphi_2(K_I K_{II}) \varphi_3(l, \bar{\Gamma}) \varphi_4 \varphi_5 \ , \quad (2)$$

K_0 – SIF which is determining in the basically conditions (without interaction of cycles, geometric singularities and different operating factors, for example, during calculations of wing lower panel thin-sheets $K_0 = \Delta\sigma\sqrt{\pi \frac{l}{2}}$; φ_1 – functional correction, which determined cycles interaction effects; φ_2 – functional correction, which depend on biaxial loads ratio, $\varphi_3(l, \bar{\Gamma})$ - functional correction on geometric singularities of element, φ_4, φ_5 – functional corrections, which estimated environmental deterioration effects.

It is rational to divide the problem of crack growth simulation, firstly, estimating inaccuracy because of real spectrum changing by program unit, and, then, estimating inaccuracy, inserted by the used model. But it is practically impossible to perform without experimental data of materials properties under cyclic loading and tests results for structure subjected random loading. Such estimation is possible to perform for specimens test under loading spectrum being not far from the real of in-service loads sequence. In fact, inaccuracy of the used method of crack growth simulation based on inserted program for acting loads modeling, mainly determined by amplitudes allocation and, in less, average value of considered process with its standard error. According to this, it is possible to use tests results for forced loading.

As material conditions seriously influenced methodical error, material design philosophy, it was necessary to choose rational test-analogue taking into account this circumstance. Before calculations for the main acting loads it can be recommended to perform test calculations for modeled loading and, if necessary, perform model correction of various parameters, included in calculating proportions.

It is should be pointed out that if changing random loading by program unit usually gives increased crack growth period, inserted in calculation process method errors , one can specify its prediction as conservative or non-conservative. Because in some cases that is possible as compensation (for example, using linier models for random loading schematization by block-programs), so summary accuracy increasing in calculations.

Cracks kinetic calculation at random stress

Great influence on crack growth rate and duration provides peak loads in random loading spectrum [3; 8]. Input of clipper factor K_{Π} (Fig. 4b) in considered process provides conservative crack growth duration estimation [8]. It was shown that crack growth maximal speed is achieved at = 2 … 2,5 [7]. Processes with such shearing ratio will be name of basic.

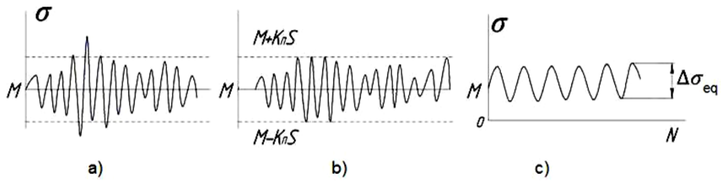


Fig. 4. Random working loading process: a - shortened process; b - M – average of random process, S – standard error, K_{Π} – clipper factor; c - equivalent harmonic loading

As basically loading modes interaction effects of cyclic loads are minimal, in Eqs. (1), (2) the functional correction $\varphi l(\mathrm{N}, l, ..)=1$. In many cases SIF K_0 is determined by the relation [5; 7; 15]

$$\mathrm{K}_0 = \varphi_{01}(6\sigma)\varphi_{02}(l) \ . \tag{3}$$

Symbol $f(l) = K_0\varphi_2(\mathrm{K_I K_{II}})\varphi_3(l,\bar{\Gamma})\varphi_4\varphi_5$ will be introduced. Then Paris-Erdohan equation is used

$$\frac{dl}{dN} = C\Delta K^n \ . \tag{4}$$

The introduced symbols of Eq. (4) can be rewritten as

$$\frac{dl}{dN} = c\varphi_{01}(\sigma)^n f^n(l) \ . \tag{5}$$

Then new variables are introduced

$$\mathrm{D} = \left[\int_{\mathrm{l_0}}^{\mathrm{l}} f^{-n}(\mathrm{x})\mathrm{dx}\right] \cdot \left[\int_{\mathrm{l_0}}^{\mathrm{l_*}} f^{-\mathrm{n}}(\mathrm{x})\mathrm{dx}\right]^{-1} \ . \tag{6}$$

In the Eq. (6) l_* is the critical length; l_0 – minimal value of crack length.

Then, Eq. (5) is transformed to

$$\dot{\mathrm{D}} = \mathrm{c}B\varphi_{01}^n(\sigma 6) \ , \tag{7}$$

where $B = \left[\int_{\mathrm{l_0}}^{\mathrm{l_*}} f^{-n}(\mathrm{x})\mathrm{dx}\right]^{-1}$.

Transformations (6), (7) are allowable, because integrals in (6) exist, and critical (allowable) length of thin-wall elements is regulated. Function *D* satisfy conditions *D(0)=0, D(t*)=1,* and agree with damage accumulation value defined according to the rule of liner damage summation (N_* number of cycles for crack growth up to critical length l_*). Introducing D is analogous to introduced by V.V. Bolotin [1] parameter for damage accumulation estimation. As followed from Eq. (7) the hypotheses of linier damages accumulation is possible in estimation crack growth kinetics and it is possible to use well-known relations for fatigue crack growth period calculations.

Let $\varphi_{01} = \Delta\sigma$, then at constant loading amplitude the crack growth period can be estimated as

$$\mathrm{N}_*\Delta\sigma 6^{\mathrm{n}} = \gamma \ , \tag{8}$$

where $\gamma = C^{-1}B^{-1}$. It is not difficult to see, that Eq. (8) is the same as with S-N curve.

As soon as for clipper factor $K_п$ = 2…2,5, main statistics of random process practically doesn't change, so for stationary narrowband loading process average durability (at positive differential at zero) is defined by equation

$$\overline{N}_{0+} = 2^{-\frac{3n}{2}}\Gamma(n/2+1)^{-1}\gamma \cdot S^{-n} . \tag{9}$$

For broadband random process the meaning of cycle is not uniformly defined, and durability calculations are related to allowable schematization methods.

If random loading process is schematized by ranges methods, then schematized density of amplitude distribution is specified by formula

$$f_{6_a}(x) = \frac{x}{\varkappa^2 S^2}\exp\left(-\frac{x^2}{2S^2\varkappa^2}\right), \tag{10}$$

and average durability, in terms of numbers of positive extremums, is given by equation

$$\overline{N}_{э+} = 2^{-\frac{3n}{2}} S^{-n}\varkappa^{-n} \cdot \Gamma^{-1}(n/2+1)\gamma. \tag{11}$$

Eq. (11) is transformed in (9) when æ = 1.

Using Eqs. (7), (8) equation for estimations of crack growth period at block-program loading modes can be introduced

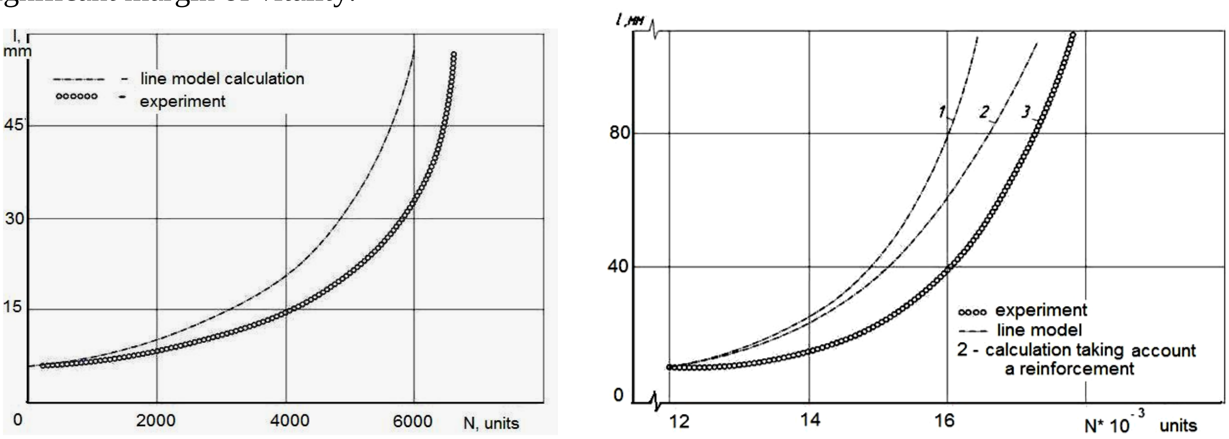
$$N_{БЛ} = \left(\frac{1}{k_1}\sum_{i=1}^{k_1} k_{3i}\Delta\sigma 6_i^n\right)^{-1}\gamma/k_2. \tag{12}$$

In Eq. (12) k_1 – number of steps in program unit, k_2 – quantity of cycles in program unit, $\Delta\sigma_i$ – the range of stress in program unit i-step, k_{3i} – number of cycles in program unit i-step.

Note, if in the Eqs. (9), (11), (12) l_{**} has variation then it is possible to have fatigue crack growth curves depended on operating time.

Fig. 5, 6 show possibility to use Eq. (12) applicably to different cases of block-programs loading.

Fig. 5a presents experimental and calculated curves of crack growth, based on program unit, which use for fatigue tests of aircraft root chord wing panel smooth sample manufactured from alloy D16chAT.Coorrelations between overload in program unit don't exceed 1,25, that's why cycles interactions effects visualize insignificantly, that confirmed by results shown in Fig. 5a. Correlation between calculated and experimental crack growth period estimations NP/NЭ ≈ 0,88, that gives insignificant margin of vitality.



a b

Fig. 5. Calculated and experimental dependences of crack length from program units during "typical flight" loading, using for aircraft wing root nervure area panel life-time tests

Fig. 5b shows calculated results for aircraft crack dangerous zone located in 10-11 nervure area in comparison with an experimental data at block-programs).

In capacity of calculating model to define corrective function φ_3, which accounts for element design philosophy, stiffened plate was accepted with width having distance between spars axis (1420 mm). Sheet thickness δ was equal to 3.5 mm, stringers step $\tau_{стр}$ = 125 mm, fasteners step $\tau_{зак}$, their diameter *d* and another geometrical adjectives are defined by proportions:

$$\frac{F_{стр}}{\delta_{стр}} = 1{,}25; \quad \frac{t}{t_{стр}} = 0{,}25; \quad \frac{d}{t} = 0{,}2. \tag{13}$$

Crack initial length was accepted equal to 10 mm, and its critical length was 110 mm. Sheets and stringers were made from material D16ATV (σ_B =460 MPa, σ_{02} = 340MPa, *E* = 73000 MPa, μ = 0,3). To describe correlation between fatigue crack growth speed and SIF range equation of Paris-Herdogan (4) was used. Parameters *C* and *m* were defined by testing results (at harmonic loading with different cycle asymmetry) of flat samples from the more resistant for cracking D16AT Al-alloy.

In SIF span calculations were accounted for stringers influence by inserting corrective function φ_3. Function φ_3 was calculated from condition of stringers-to-skin resilient fastening. Function φ_3 calculated results are shown in table 1. Calculation was performed in two variants: plate with account stiffening by stringers and without stringers influence.

Modified results graphical interpretation has shown possibility of fatigue crack growth speed calculations by linear model. At this, given estimations of life-time period have acceptable reserve. (1 case: $N_p/N_Э$ = 0,776, 2 case: $N_p/N_Э$ = 0,928). It should be also noted that life-time period estimation accuracy materially increases (≈ 20% up) in case of influence on the fatigue crack kinetic of stiffener elements (stringers).

Fig. 6 illustrates application proportion possibility (12) for life-time period estimations at loading by “Twist” type program unit (Fig. 1f). Results analysis shows, that using linear model in calculations is the comfort method of life-time period estimation, because inaccuracy, received in results, gives some vitality margin ($N_p/N_Э$ = 0,806).

Table 1

Corrective function φ_3 dependence on crack length

l, мм	10	20	30	40	50	80	110
φ3	0,992	0,97	0,943	0,917	0,893	0,843	0,81

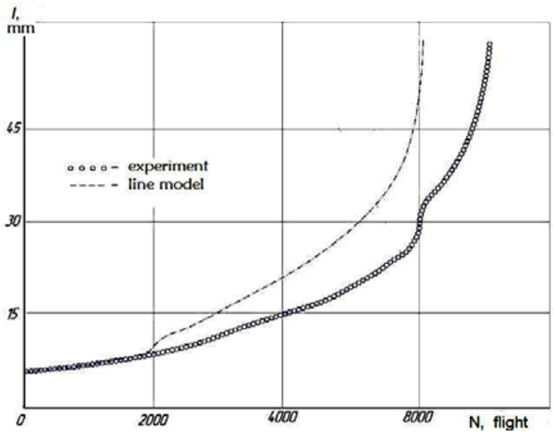


Fig. 6. Results of calculated-experimental cracks kinetic estimation at loading by program unit “TWIST”

Equivalent span tension estimation

As soon as in stationary loading Gauss processes influence conditions during base modes cycle interaction minimizing, and curves, which show crack length dependence on cycles quantity or time, are smooth, it is possible to declare allowance of base process modeling by harmonic loading with tension span $\Delta\sigma_{eqv}$ (Fig. 4c). At this, crack growth duration calculations are carried out by cracks kinetic linear equations Paris-Herdogan type.

Using hypothesis of damages linear summering, it is possible to write

$$\bar{n}_{\Delta} \int_0^{\infty} \frac{f(\Delta\sigma) d\Delta\sigma}{N_*(\Delta\sigma)} = \frac{\bar{n}_{\Pi}}{N_*(\Delta\sigma_{eqv})}, \qquad (14)$$

where $f(\Delta\sigma_{eqv})$ - density of tension span distribution at chosen process schematization method, $N_*(\Delta\sigma_{eqv})$ - loading cycles quantity up to fracture at tension span $\Delta\sigma_{eqv}$, n_{Δ} - frequency of loading operation mode (quantity of zero per time unit, extremums quantity, full cycles quantity etc), $n_{п}$ - harmonic tension equivalent frequency. Inserting proportion (8) to (14), receive

$$\bar{n}_{\Delta} \int_0^{\infty} \Delta\sigma^n f(\Delta\sigma) = \bar{n}_{\Pi} \Delta\sigma_{eqv} \qquad (15)$$

or

$$\bar{n}_{\Pi} \Delta\sigma_{eqv}^n = \bar{n}_{\Delta} \langle \Delta\sigma^n \rangle. \qquad (16)$$

Proportions (15) and (16) depend on three parameters n, n_{Δ} и n_{Π},, that because their choice is mainly defined according to the degree of equivalent harmonic and operational loading.

Examine a particular case of formula (16). Let the process schematize by spans method. In this case of cycles frequency n_{Δ} accords working loading maximum frequency. Let chose a frequency n_n equal to n_{0+} crossings of average load level random process with positive derivative. In this case proportion (16) takes view

$$\Delta\sigma_{eqv} = \varkappa^{-\frac{1}{n}} \sqrt[n]{\langle \Delta\sigma^n \rangle}. \qquad (17)$$

For stationary Gauss process at $n = 2$ independently from irregularity ratio $\Delta\sigma_{eqv} = 2\sqrt{2}S$.

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ИСПОЛЬЗОВАНИЕ ЛИНЕЙНЫХ МОДЕЛЕЙ В РАСЧЕТАХ ДЛИТЕЛЬНОСТИ РОСТА ТРЕЩИН ПРИ СЛУЧАЙНЫХ НАГРУЗКАХ

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В работе приводятся экспериментальные и теоретические результаты исследований в области оценок длительности роста трещин в условиях воздействия нерегулярных нагрузок, имитирующих эксплуатационные. Показана возможность применения моделей типа Пэриса-Эрдогана для расчетов периода живучести тонкостенных элементов авиаконструкций. Установлена аналогия в расчетах долговечности и расчетах живучести. Вводится понятие меры повреждений для оценки возможности применения линейной модели накопления повреждений в расчетах длительности роста трещин. Теоретические положения сопоставляются с результатами экспериментальных исследований трещиностойкости плоских образцов-пластин из сплавов Д16АТ (аналог 2024-Т3) и В95АТВ (аналог 7075-Т6). Нагружение в эксперименте представляло собой стационарные гауссовские процессы.

Ключевые слова: длительность роста трещин; нагрузки с переменной амплитудой, усталостная трещина, алюминиевые сплавы

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