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2.5.14 – Прочность и тепловые режимы летательных аппаратов;
2.5.15 – Тепловые электроракетные двигатели и энергоустановки летательных аппаратов;
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Features of vortex trace propagation for aircraft with propellers

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Abstract: The article presents the results of a study of the characteristics of the wake vortex of aircraft with turboprop engines. Using the example of the An-12 aircraft, it is shown that rotating propellers make a noticeable contribution to the propagation of the vortex trail behind the aircraft. This is proved by some studies, as well as numerous observations. It also describes a technique for studying the wake vortex of aircraft with propellers. The method is based on the method of discrete vortices. The relevance of such studies is due to the growing interest of carrier companies in aircraft with turboprop engines. It has been proven that when transporting passengers and cargo on such vessels over distances of 700–800 km, maintenance and fuel costs are reduced by about 30–40%. Therefore, the fleet of turboprop aircraft, such as An-22, An-70, An-12, as well as Tu-95, Il-38, C-130, etc., has been preserved so far. New turboprop aircraft are being developed and put into operation: A-400M, Il-114, Il-112M. The vortex trail behind such aircraft also poses a danger to other aircraft flying behind. A feature of the propagation of the wake vortex behind aircraft with propellers is the interaction of vortices coming off the airframe and vortices from the propellers. As a result, due to the rotation of all the screws in one direction, symmetry is broken in the propagation of vortices descending from the right and left halves of the wing. Therefore, it is important to understand how differently the vortices that descend from the airframe of an aircraft with turboprop engines behave. For the convenience of the study, the method of accounting for the effect of vortices from screws is integrated into a special calculation and software package, also based on the method of discrete vortices. In it, when calculating the characteristics of the wake vortex, the flight weight, speed and altitude of the aircraft, its flight configuration, atmospheric conditions, proximity of the earth, axial velocity in the core of the vortex and some other factors are taken into account. This complex has passed the necessary testing and state registration. A number of measures were carried out to validate and verify the developed complex, confirming the operability of the programs included in it and the reliability of the results obtained from it. The results of the study of the characteristics of the wake vortex behind the Antonov-12 aircraft in the form of vertical velocity spectra and fields of perturbed velocities at various distances from it are presented. It is shown that propellers noticeably affect the propagation of the wake vortex behind turboprop aircraft. This circumstance must be taken into account by the crews of aircraft flying behind such aircraft.

Key words: propellers, wake vortex, aircraft, vortex interaction, turboprop aircraft.

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Особенности распространения вихревого следа за воздушными судами с винтами

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Аннотация: В статье приводятся результаты исследования характеристик вихревого следа за воздушными судами с турбовинтовыми двигателями. На примере самолета Ан-12 показано, что вращающиеся винты вносят заметный вклад в распространение вихревого следа за самолетом. Это доказывают и некоторые исследования, а также многочисленные наблюдения. Также описывается методика для исследования вихревого следа за воздушными судами с винтами. В основе методики лежит метод дискретных вихрей. Актуальность таких исследований обусловлена возрастающим интересом компаний-перевозчиков к воздушным судам с турбовинтовыми двигателями. Доказано, что при перевозках пассажиров и грузов на таких судах на расстояния 700–800 км затраты по обслуживанию и на топливо сокращаются примерно на 30–40 %. Поэтому до сих пор сохранен парк турбовинтовых самолетов, таких как Ан-22, Ан-70, Ан-12, а также Ту-95, Ил-38, С-130 и др. Разрабатываются и вводятся в эксплуатацию новые турбовинтовые самолеты: А-400М, Ил-114, Ил-112М. Вихревой след за такими самолетами также представляет опасность для других, летящих следом самолетов. Особенностью распространения вихревого следа за самолетами с винтами является взаимодействие вихрей, сходящих с планера самолета и вихрей от винтов. В результате из-за вращения всех винтов в одну сторону нарушается симметрия в распространении вихрей, сходящих с правой и левой половин крыла. Поэтому важно понимать, насколько по-разному ведут себя вихри, сходящие с планера самолета с турбовинтовыми двигателями. Для удобства исследования методика учета влияния вихрей от винтов интегрирована в специальный расчетно-программный комплекс, базирующийся также на методе дискретных вихрей. В нем при расчете характеристик вихревого следа учитываются полетный вес, скорость и высота полета самолета, его полетная конфигурация, атмосферные условия, близость земли, осевая скорость в ядре вихря и некоторые другие факторы. Этот комплекс прошел необходимую апробацию и государственную регистрацию. Был выполнен ряд мероприятий по валидации и верификации разработанного комплекса, подтверждающих работоспособность программ, входящих в него, и достоверность получаемых по нему результатов. Приводятся результаты исследования характеристик вихревого следа за самолетом Ан-12 в виде спектров вертикальной скорости и полей возмущенных скоростей на различных удалениях от него. Показано, что воздушные винты заметно влияют на распространение вихревого следа за турбовинтовыми самолетами. Это обстоятельство необходимо учитывать экипажам воздушных судов, летящих следом за такими самолетами.

Ключевые слова: воздушные винты, вихревой след, воздушное судно, взаимодействие вихрей, турбовинтовые самолеты.

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Introduction

Nowadays the aeronautical communities of many countries face the current problem of the ever-growing airport capacity provision due to air traffic increase maintaining the objective aircraft flight safety level. Vortex safety provision [1–5] is one of main challenges for implementation of such plans. The essence of vortex safety issue is wake vortex following the aircraft [6–11]. This wake is an induced velocity and pressure field which is dangerous for aircraft following it. It is worth noticing that one should distinguish between the concepts of wake vortex and vortex path. It is correctly suggested in work [12] that there is a *wake vortex* following the body in motion developing lift (for example, an aircraft). Whether the body in motion does not develop lift (for example, a car), there is a *vortex path* following it.

The work focuses on a wake vortex following the propeller aircraft. Turboprop aircraft observation shows us that wake vortex following them is different from the one following the turbo-jet aircraft (fig. 1). It is connected to propeller rotation influencing the aircraft wake vortex. Wake vortex following the aircraft loses its symmetry almost at once as the propeller spins one way, which can be seen in (fig. 1).

The long-haul propeller aircraft introduction has required the research of the long-distance wake vortices following them. Wake vortex following such aircraft is also dangerous for other aircraft behind it. The question of propeller impact on long-distance wake vortex characteristics is still open so far. Analysis shows us that the research developments in this area are insufficient. The majority of them are scattered studies in the flight experiment of the USA Department of Transportation program on wake vortex following the propeller aircraft. There are almost no approaches and mathematical models for wake vortex following the propeller aircraft.

The interest in propeller aircraft has grown recently, as they are cheaper in terms of passenger and cargo transportation on equal distances, in comparison to turbo-jet aircraft. Some foreign experts estimate that service and fuel charges are reduced by about 30–40% during passenger and cargo transportation given the distance of

Fig. 1. Random visualization of the wake vortex following the Antonov-12 turboprop aircraft during takeoff

700–800 km. That is why some aerospace corporations are starting the turboprop aircraft development. For instance, the Canadian engineering company Bombardier is now developing and producing the twin-engine turboprop aircraft DHC-8. Airbus Military has developed the A-400M aircraft and started its manufacturing production. ATR is doing the same thing. There are also Ilyushin Il-114 and Ilyushin Il-112B in use in Russia. There is also data about other similar constructions.

Research methodology

Wake vortex following the propeller aircraft research methodology, used in this work, is described in details in paper [12] and article [13]. In this article it is integrated into a special calculating and software package [14], also based on discrete wake method [15–17]. The essence of integration is in the following. It was necessary to develop such a propeller mathematic model, in which its work effect record was made through discrete vortex points with the known circulations and coordinates on Trefftz plane. The fact is the long-distance wake vortex mathematical model of the calculating and software package is also based on vortex points. In this case the propeller mathematical model is integrated into the long-distance wake vortex mathematic model [12, 18].

Let us interpolate the following designations:

d – propeller diameter;

L – the aircraft typical linear dimension;

ω – the propeller angular velocity;

V_0 – the airspeed;

r_0 – the propeller rotor head radius;

$\xi = \dfrac{r_0}{R}$ – the relative propeller rotor head radius; where $R = \dfrac{d}{2}$;

$\overline{\alpha}$ – the propeller thrust coefficient;

n – the number of propeller blades;

$\dfrac{V_0}{nd} = \lambda$ – the propeller speed coefficient;

$\overline{\beta}$ – the propeller power coefficient.

The following vortex model of the propeller (fig. 2) is developed for the given mathematical model integration into the calculating and software package [14]. There is an axial flow circulation wake Γ^* in the middle of the propeller, the n wakes are set around the propeller circumference perimeter, modelling the propeller jet flow. The research in work [13] showed us, that n should correspond to the number of propeller blades. Then the wake circulation around the propeller circumference will be equal to Γ^*/n. It is possible to define the intensity of the propeller-generated axial flow wake by formula [13], whether the propeller work regime is set – λ, $\overline{\alpha}$, $\overline{\beta}$ and the relative propeller rotor head diameter is known:

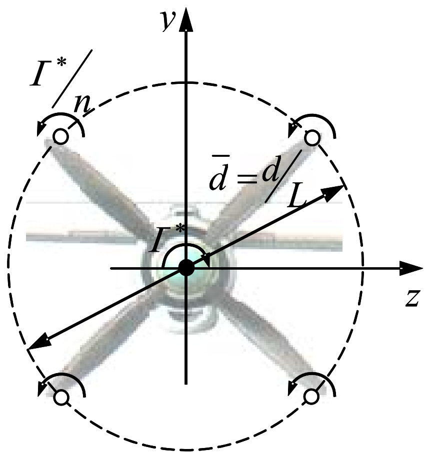

Fig. 2. Vortex model of the propeller screw

$$\overline{\Gamma}=\frac{4\overline{\beta}}{\pi^3\left(1-\xi^2\right)\left[\frac{\lambda}{2\pi}+\sqrt{\frac{\lambda^2}{4\pi^2}+\frac{2\overline{\alpha}}{\pi^3\left(1-\xi^2\right)}}\right]}.$$

Let us interpolate the axial flow non-dimensional circulation according to formulae for the aircraft in general, $\Gamma^*=\frac{\Gamma}{V_0 L}$, where L is a typical size, then $\overline{\Gamma}$ and Γ^* will be linked by formula

$$\overline{\Gamma}\omega Rd=\Gamma^* V_0 L$$

where

$$\Gamma^*=\overline{\Gamma}\frac{\omega Rd}{V_0 L}=\overline{\Gamma}\frac{2\pi nd}{V_0}\frac{d}{2L}=\overline{\Gamma}\frac{\pi}{\lambda}\overline{d},$$

and finally

$$\Gamma^*=\overline{\Gamma}\frac{\pi}{\lambda}\overline{d}.$$

$\overline{d}$ is a relative propeller diameter $\overline{d}=d/L$. Then, the vortex propeller jet flow scheme (in Trefftz plane) will look as it is shown in Figure 1: n number of vortices, set around the circumference by the diameter equal to the propeller diameter, model the propeller jet flow surface. The number of vortices corresponds to the number of blades here. Circulation of each vortex is Γ^*/n, and spinning direction opposes the axial flow vortex spinning direction. At the same time the axial flow vortex produces spinning, which corresponds to propeller spinning direction. Thus, the purpose is achieved. The vortex points, which are modelling the propeller work, are integrated into the calculating and software package [14].

The results of the research

The characteristics of long-distance wake vortex following the C-130 aircraft at 1000 m height, at V = 51 m/s speed were calculated to confirm the effectiveness of the developed methodology and credibility of the results based on them. The flight experiment data has been obtained from paper [2] on wake vortex maximum vertical velocity measurement for this aircraft and the flight conditions. There are the vertical speed calculations behind the C-130 at distances X = 0 and 1.4 km in Figure 2. It can be seen that the vertical speed graph is sawtooth if X = 0 (that is fuselage longitudinal section, rhombs). It is connected with propeller rotation impact on the wake vortex behind the aircraft.

The whole spectrum of vertical velocity (squares) is calculated at distance X = 1.4 km from C-130 aircraft. It can be seen that the calculation (squares) and flight experiment (triangles) correspond satisfactorily to each other, which confirms indirectly the credibility of the results (fig. 3).

Furthermore, the characteristics of Antonov An-12 aircraft wake vortex were also observed. It is shown that rotating propellers cause a noticeable impact on wake vortex distribution. The first stage shows us, how the vertical velocity spectrum changes in the middle of An-12 vortex without taking propeller spinning into considera-

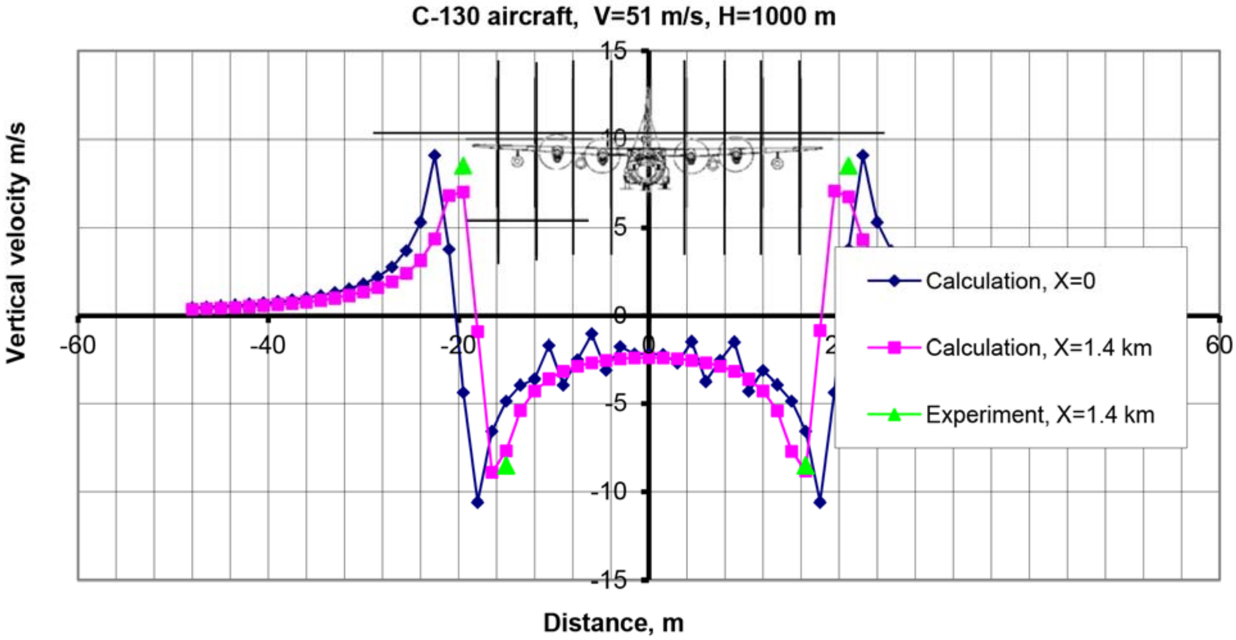

Fig. 3. Vertical velocity distribution in the vortex core of the C-130 aircraft

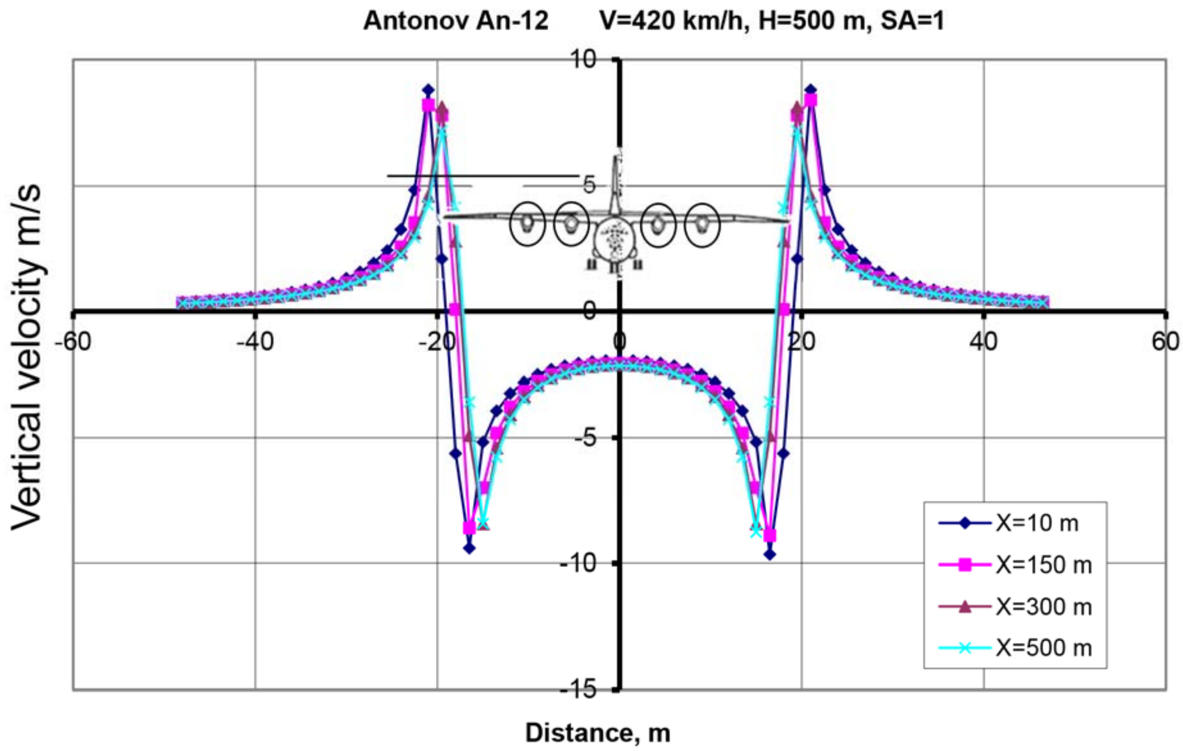

Fig. 4. Distribution of vertical velocities in the vortex core of the An-12 aircraft (excluding the influence of propellers)

tion (fig. 4), and with it (fig. 5). The vertical velocity was calculated at X = 10, 150, 300 и 500 m distance. The speed of flight then was V = 420 km/h, and height was H = 500 m. λ, $\overline{\alpha}$, $\overline{\beta}$ parameters were extracted from An-12 cruise flight diagram. The atmosphere is stable, SA = 1 [12].

It can be seen that vertical velocity spectra are significantly different in Figures 4 and 5. It is also

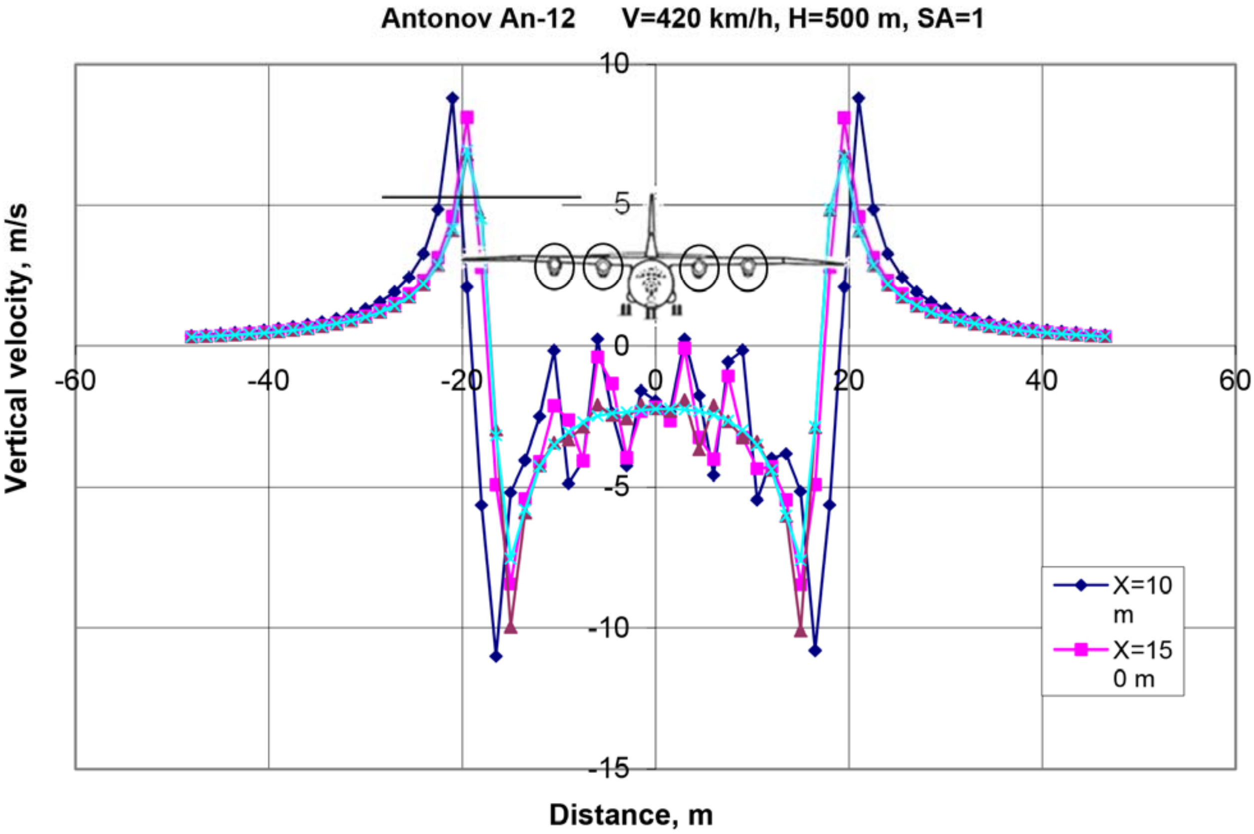

Fig. 5. Distribution of vertical velocities in the vortex core of the An-12 aircraft
(Taking into account the influence of propellers)

worth noticing, that propeller rotation impact on vertical velocity almost disappears already at X = 500 m distance. It is connected with vortex natural ease-off due to atmosphere turbulence, along with vortex dissipation and diffusion.

At the second stage, the perturbed velocity fields behind the An-12 at up to 2 km distance (fig. 6) were observed. There are the results of wake vortex characteristics calculation as the perturbed velocity fields, which are presented as vectors of mixed speed W, calculated by formula

$$W = \sqrt{W_z^2 + W_y^2}$$

W_z and W_y are the parts of the vertical and horizontal perturbed speed. W speed variable can also be easily determined through step scale in Figure 6, which size is 10 m/s.

It can be seen that the vortex symmetry from the left and the right wing is broken while X distance from the aircraft is increasing.

This circumstance drastically distinguishes the wake vortex behind the turboprop and turbo-jet aircraft. The wake vortex following the turbo-jet aircraft remains symmetrical for a long time on both the left and the right wings [2, 12, 19–26]. This symmetry is broken almost at once between the turboprop aircraft due to propeller spinning impact. There are the works [12, 27], in which it is shown that propellers spinning one way also cause impact on the aircraft aerodynamic characteristics. It is connected with non-symmetrical flowing around of the aircraft airframe. There is some yet noticeable yawing and roll during the turboprop flight. There are some special procedures implemented in some aircraft structure for their disposal. Nevertheless, it can lead to increase in drag, and, consequently, to extra fuel costs. There are the aircraft with left and right propellers spinning different ways which allows to dispose airframe non-symmetrical flowing around. For instance, A-400M by Airbus Military.

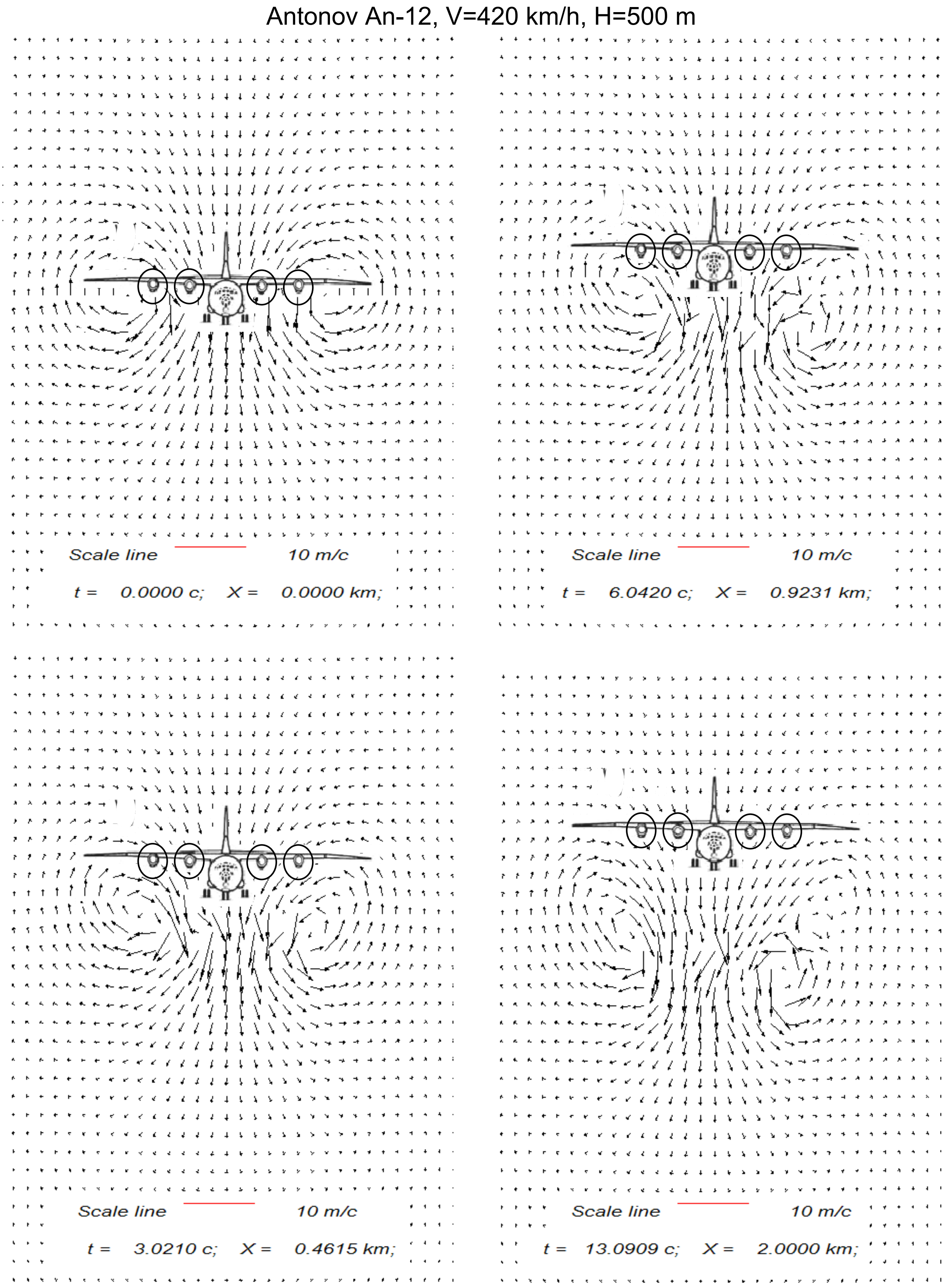

Fig. 6. The field of perturbed velocities behind the An-12 aircraft at various distances

Conclusions

Thus, the calculation showed us that wake vortex behind the turboprop aircraft differs drastically from the one behind the turbo-jet aircraft. The reason of such a difference is propeller rotation. Propellers of almost all the used turboprop aircraft rotate one way. Wake vortex symmetry behind the aircraft is broken during propeller vortex interaction with vortices from the aircraft airframe. It is necessary for the crews of aircraft following the turboprop aircraft to consider this circumstance. Besides that, it is also necessary to consider this peculiarity while providing wake vortex safety in the vicinity of large airports, when the safe separation between the taking-off and landing aircraft should be maintained.

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