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## MATHEMATICAL MODELING OF THE PROCESS OF FUNCTIONING OF OBJECTS AND TECHNICAL MEANS OF ENSURING AIRFIELD CONTROL

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### ABSTRACT

In the process of performing a complex of works on refueling of civil aviation aircraft, one of the key issues is to ensure flight safety by controlling the quality of aviation fuel directly during refueling operations. Currently, to ensure the purity of the refueled jet fuel, water separators with filter elements of a normalized degree of purification are installed on the aircraft refueling facilities, the operation of which in the working area provides normalized indicators of cleaning jet fuel from water and mechanical impurities. As practice shows, in the process of refueling aircraft, for various objective and subjective reasons, sometimes there are stochastic situations in which quality indicators go beyond the limits established by regulatory documentation and are not deterministic, and the subsequent state of such a system is described by values that characterize an extremely low level of jet fuel purification with negative consequences for flight safety. This paper presents a mathematical description of the functioning of water separator filters in the working area, where standardized indicators of the quality of aviation fuel are provided during the refueling of aircraft. The article deals with the issue of blocking the refueling of aircraft in the event of the appearance of non-normalized technical documentation indicators of the quality of aviation fuel, which arise due to a number of different factors that lead to negative cause-and-effect relationships for flight safety. Based on the mathematical description, an approach to creating a system for protecting and blocking the refueling process under the working name "Barrier" is proposed. Of the greatest interest for the study are typical water separator filters installed on refueling vehicles as terminal technical devices for fuel purification during refueling of aircraft.

**Key words:** filter water separator, filter element, cleaning products, normalized and non-normalized quality indicators, working area, mathematical field, pressure drop, mathematical model, mathematical software, least squares method, working fluid of jet and turbojet engines of aircraft.

### INTRODUCTION

The subject of study in the present paper is the working area of the filter elements of the filter water separators as the jet fuel cleaning products<sup>1</sup>, installed at refueling facilities as the terminal technical facilities adjacent to the aircraft tanks<sup>2</sup>.

The purpose of the research is to develop the basics of the mathematical model of system for protecting the refueling facilities from the fuel quality indicators exceeding the working area, both by pressure drop  $\Delta P$ , and consumption  $Q$  based on manufacturer's technical documentation for the cleaning products.

The authors have set the problem – to use the suggested mathematical description in the mathematical software of the on-board digital controller capable of blocking the refueling process while meeting the fuel quality stochastic criteria, as working fluid of jet and turbojet aircraft engines [1].

*The working hypothesis* is accepted in the research, as a statistically distributed proposal about the difference between the mathematical fields [2] of non- and working area of filter elements, due to the standardized diff of the fuel quality indicators, divided by the aircraft refueling process blocking

<sup>1</sup> GOST R 18.12.03-2018. (2018). Technology of the Aviation Fueling. Filtering Equipment in Aviation Fuel Supply. General Technical Requirements. Moscow: Standartinform, 23 p. (in Russian).

<sup>2</sup> ICAO Doc 9859. (2018). Safety Management Manual. 4th ed. Montreal, ICAO, 218 p.

borders, along with the ways of solving the given problem due to the hypothetic connection of the research subject and the proposed actions.

Statement of the research problem: to give the mathematical description of the working areas of terminal fuel cleaning products filter elements while functioning in the working zone as the predictive process, and as the random and unpredictable process forming the borderline of blocking the aircraft refueling process in order to distinguish them. In terms of control optimization – this is the area of boundaries of derivation of the signals for the refueling process rejection.

The relevance of the following topic is in the necessity of developing the algorithms, as the set of instructions, based on the revealed correspondence of the processes under investigation and the description of the input and output data.

The input data must be entered into the filter elements as the technical data and be used for solving the given problems. *The output data* must be presented as the outgoing signal for the refueling process rejection, as footage on the refueling facilities controller display and as a piece of information in database.

## DISCUSSION OF TEST DATA

Let us present the standard limited pressure drop  $\Delta P$  dependence of fuel consumption  $Q$ , which is processed for analysis [3, 4] and attached to the batch of filter elements by the manufacturer, in order to carry out the research of the predictive process and describe the mathematical relationship.

On account of the data for the research, the working area is a multitude of freehand curves, let us consider the processes of fuel cleaning in that area as the *predictive* ones, and describe them with explicit dependences. Let us use the method of straight-line approximation, which allows us to study the numerical characteristics, along with the qualitative behavior of the subject of the research (fig. 1), for the mathematical description of the processes in the working area.

Let us use the technical documentation data of the filter element factory of origin<sup>3</sup>, in order to study the function by the straight-line approximation method [5, 6]. The working area is in the set of the following  $\alpha$  and  $b$  points of the simple equations:

1. For the maximum limit: 0.95...2.0;
2. For the minimum limit: 0.06...0.3.

As the current valuations of the cleaning products parameters may be present in any valuation in the working area, let us use the equations of line function, for the aircraft fueler onboard controller software, which we will present in the form of [7]:

$$\gamma = \alpha \cdot x + b \quad (1)$$

Thus, the solution of the equation is the searching such  $\alpha$  and  $b$  coefficients, that all the required working area points were most closely to the approximating line.

Let us use the least squares method, its content is minimization of sum of squared deviations of some functions from the required variables, meaning that the sum of squared deviations of the point valuations from the approximating line is present in the minimal valuation [8–10], meaning that:

$$F(\alpha, b) = \sum_{i=1}^n (\gamma_i - (\alpha \cdot x + b))^2 \rightarrow \min \quad (2)$$

Solving on the given problem is in finding the extreme point of function of the two variables.

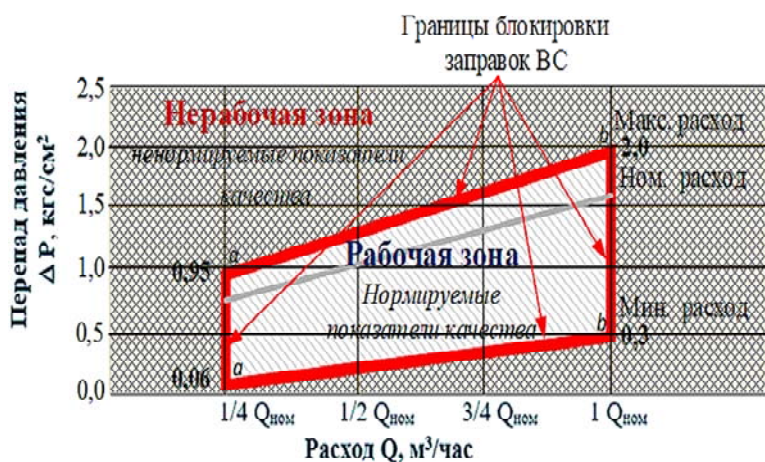
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<sup>3</sup> Filters-water separators for fuel horizontal FVGk and FVGk-U. The unit is a scientific and production association. Research and testing, development and manufacture, installation and commissioning of fuel supply equipment and filtration equipment. Available at: <http://www.agregatnp.ru/production/korpusy/fvg.html> (accessed: 10.04.2021 ). (in Russian)

It is clear from the noted below Figure 1, that the points  $\alpha$  and  $b$  we are interested in are in the working area. Thus, the solution of the integral function (1, 2) will be in following expression in general terms:

$$\int_{\alpha}^{\beta} f(x)dx, \quad (3)$$

where:  $\alpha$  and  $\beta$  are the limits of integral.



Terms:

Границы блокировки заправки – aircraft refueling blocking boundaries,  
Нерабочая зона – non-working area,  
Ненормируемые показатели качества – non-regulated quality indexes,  
Рабочая зона – working area,  
Макс. расход – Maximum consumption,  
Ном. расход – Nominal consumption,  
Мин. расход – Minimum consumption,  
Расход  $Q$ , м³/час – Consumption m³/hr,  
Перепад давления  $\Delta P$  кгс/см² – Pressure drop  $\Delta P$  kp/cm²

Fig. 1. Typical dependence of the maximum allowable pressure drop  $\Delta P$  on the jet fuel consumption  $Q$

The software of data for limiting pressure drop in the working zone is based on these mathematical conclusions, the results are presented as the operating efficiency freehand curves. The signal of blocking the aircraft refueling is formed by hitting the limited valuations of consumption and pressure drop parameters.

There is also a fragment of monitor and a line of forming the optimized signal of maximum continuous consumption of facilities of filtration (the grey line) (fig. 2). The consumption and pressure drop optimization is carried out in order to increase the filter element resource up to 30% of the maximum consumption regime.

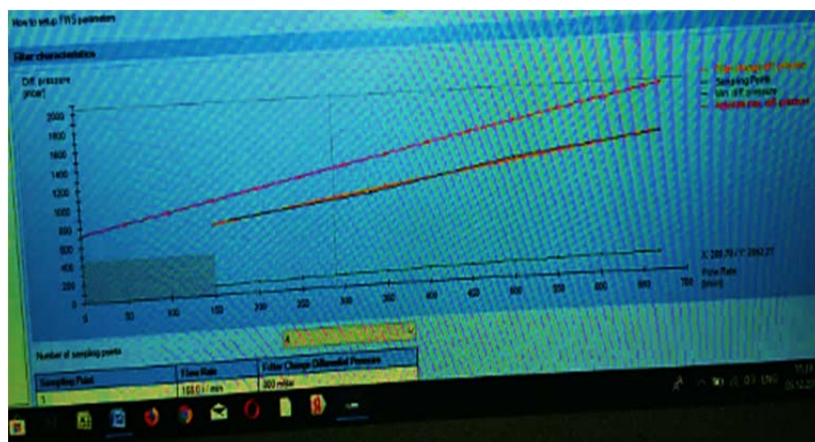


Fig. 2. A fragment of the monitor with the calculated performance characteristics of the filter elements of the water separator filter

The optimization of the parameters of the facilities of filtration is also crucial for choosing the power unit of the aircraft refueling vehicles within design process, because such an approach allows us to use both the resource of the power unit itself and the resources of torque forwarding mechanism for the technologic compartment of the aircraft refueling vehicle in the optimal way while refueling the aircraft.

However, the fuel quality indicators exceeding the working area is probable in any direction while servicing by objective and subjective reasons, in other words, there is an opportunity of transition into nondeterministic or stochastic (random) processes. It makes sense to present such processes as a stochastic matrix, by which we will describe the working regimes of the facilities of filtration beyond the working area and input the results for further software calculation (fig. 3).

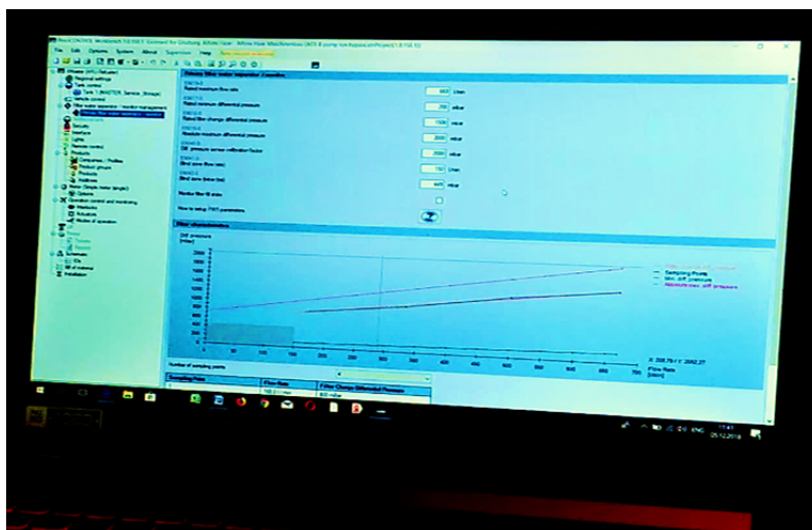


Fig. 3. Input of the data array on the pressure drop limits according to the technical documentation for the filter element

As it is known, the stochastic matrix is the matrix of transition probability as the starting point for the theory of stochastic processes [4] in Markovian network [11, 12]. In this case the matrix  $P = (P_{ij})$ , where  $i$  и  $j = 1, 2 \dots$  is stochastic and is [13]:

$$P_{ij} \geq 0, \quad \forall i, j = 1, 2 \dots \text{и} \sum_{j=1}^{\infty} P_{ij} = 1 \quad \forall i, \quad (4)$$

where:  $\forall$  – mathematical sign For All.

According to that we find the discriminant of the algebraic numeric field [3, 14, 15] of the working area and its numeric invariant, we will use its feature of changing the size of the algebraic numeric field in order to form the signals of managing the facilities of filtration with the opposite sign. Thus, the research showed us, that, the following discriminant of the numeric field beyond the working area is in proportion to square number of the working area and provides the branch of the prime numbers. The further invariant development [16] shows us, that, if roof equations in the working area are in the basis of the stochastic matrix, the rest of the valuations will be different in terms of the sign and value.

## RESEARCH RESULTS APPLICATION

The next step is uploading of the mathematic software into the onboard estimated data digital controller for the filter elements in the fuel cleaning facilities. Let us notice in the process, that while the periodical change of the filter elements, it is necessary to pay attention to the characteristics of their

working area, and in case of divergence detection it is necessary to reformat the mathematical software into the experimental data, otherwise the "Barrier" system of protection from insertion of the non-standard fuel will work incorrectly.

The procedure of data input into the onboard digital controller is exactly an engineering technological operation with the help of USB-technologies [17, 18] and is not of academic novelty, nevertheless the authors have seen fit in mentioning about it here.

## CONCLUSIONS

According to the research results, let us make a conclusion, as the exceeding the working area by the fuel cleaning products quality indicators while refueling the aircraft is the crucial factor, there is an opportunity of forming the signal of rejection of the aircraft refueling operations for the onboard digital controllers, by using the results of mathematical modelling of process of forming the filter element working area, which will become a kind of a barrier from insertion of the non-standard fuel into the aircraft tankage.

## REFERENCES

1. **Bordunov, V.D. and Eliseev, B.P.** (2015). *The legal policy strategy of the application 19 of annex "safety management" practice*. Nauchnyy Vestnik MGTU GA, no. 216, pp. 5–10. (in Russian)
2. **Burbaki, N.** (1965). *Algebra. Chast 2: Mnogochleny i polya. Uporyadochennyye gruppy: Monografiya* [Algebra. Part 2. Polynomials and fields. Ordered groups: Monograph]. Translated from French by V.Ye. Govorov, Yu.I. Manin, A.V. Mikhalev and others, in Yu.I. Manin (Ed.). Moscow: Nauka, 298 p. (in Russian)
3. **Brailko, A.A.** (2018). *Metod nepreryvnogo monitoringa chistoty aviato-pliva v tekhnologicheskoy skheme toplivoobespecheniya vozdukhnykh sudov: dis. ... kand. tekhn. nauk* [Method of continuous monitoring of aviation fuel purity in the technological scheme of aircraft fuel supply. Dissertation for the Candidate of Technical Sciences]. Moscow: MGTU GA, 134 p. (in Russian)
4. **Rybakov, K.V.** (1982). *Aviatsionnyye filtry dlya topliv, masel, gidravlicheskiykh zhidkostey i vozdukh: uchebnoye posobiye* [Aviation filters for fuels, oils, hydraulic fluids and air: Tutorial]. Moscow: Mashinostroyeniye, 103 p. (in Russian)
5. **Vinogradov, V.N., Gay, E.V. and Rabotnov, N.S.** (1987). *Analiticheskaya approksimatsiya dannykh v yadernoy i neytronnoy fizike* [Analytical approximation of data in nuclear and neutron physics]. Moscow: Energoatomizdat, 128 p. (in Russian)
6. **Laurent, P.J.** (1975). *Approksimatsiya i optimizatsiya* [Approximation and optimization]. Translated from French by Yu.S. Zavyalov, R.A. Zvyagina, B.I. Kvasov, B.I. Shmyrev, in G.Sh. Rubinshteyn, N.N. Yanenko (Eds.). Moscow: Mir, 496 p. (in Russian)
7. **Magnus, Ya.R., Katyshev, P.K. and Peresetsky, A.A.** (2007). *Ekonometrika. Nachalnyy kurs: uchebnoye posobiye* [Econometrics. Initial course: Tutorial]. 8th ed., ispravlennoye. Moscow: Delo, 504 p. (in Russian)
8. **Ayvazyan, S.A.** (2001). *Prikladnaya statistika. Osnovy ekonometriki. Tom 2* [Applied statistics. Fundamentals of econometrics. Vol. 2]. Moscow: Uniti-Dana, 432 p. (in Russian)
9. **Vinogradov, I.M. (ed.).** (1979). *Matematicheskaya entsiklopediya. V 5 tomakh. Tom 2: D'Alemberta operator – Kooperativnaya igra* [Encyclopedia of Mathematics. In 5 vols. Vol. 2: D'Alembert operator – Cooperative game]. Moscow: Sovetskaya Entsiklopediya, 552 p. (in Russian)
10. **Mitin, I.V. and Rusakov, V.S.** (2012). *Analiz i obrabotka eksperimentalnykh dannykh: uchebnoye posobiye* [Experiment data analysis and processing: Tutorial]. 5th ed. Moscow: MGU, 44 p. (in Russian)

11. Kelbert, M.Ya. and Sukhov, Yu.M. (2010). *Veroyatnost i statistika v primerakh i zadachakh. Tom 2: Markovskiye tsepi kak otpravnaya tochka teorii sluchaynykh protsessov i ikh prilozheniya* [Probability and statistics in examples and problems. Vol. 2: Markov chains as a starting point in the theory of random processes and their applications]. Moscow: MTSNMO, 295 p. (in Russian)
12. Markov, A.A. (1906). *Rasprostraneniye zakona bolshikh chisel na velichiny, zavisyashchiye drug ot druga* [Extension of the large numbers law to quantities that depend on each other]. *Izvestiya fiziko-matematicheskogo obshchestva pri Kazanskom universitete*, 2nd series, vol. 15, pp. 135–156. (in Russian)
13. Landau, E. (2010). *Osnovy analiza. Deystviya nad tselyimi, ratsionalnymi, irratsionalnymi, kompleksnymi chislami. Dopolneniye k uchebnikam po differentsialnomu i integralnomu ischisleniyu* [Fundamentals of analysis. Actions on integers, rational, irrational, complex numbers. Supplement to textbooks on differential and integral calculus]. 3rd ed. Moscow: KomKniga, 184 p. (in Russian)
14. Burbaki, N. (1963). *Ocherki po istorii matematiki* [Essays on the history of mathematics]. Moscow: Izdatelstvo inostrannoy literatury, 292 p. (in Russian)
15. Bourbaki, N. (1994). *Elements of the history of mathematics*. Springer, Berlin, Heidelberg, 301 p. DOI: 10.1007/978-3-642-61693-8
16. Diedennone, J., Kerrol, J. and Mumford, D. (1974). *Geometricheskaya teoriya invariantov* [Geometric theory of invariants]. Moscow: Mir, 278 p. (in Russian)
17. Mueller, S. (2007). *Upgrading and Repairing PCs*. 17th ed. Moscow: Williams, 1504 p. (in Russian)
18. Hultzebosch, Yu. (2009). *USB v elektronike* [USB in electronics]. 2nd ed., ispravlennoye. St. Petersburg: BKHV-Petersburg, 224 p. (in Russian)

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#### ИССЛЕДОВАНИЕ РАБОЧЕЙ ЗОНЫ СРЕДСТВ ОЧИСТКИ АВИАТОПЛИВА ПРИ ВЫПОЛНЕНИИ ЗАПРАВОЧНЫХ ОПЕРАЦИЙ ВОЗДУШНЫХ СУДОВ

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В процессе выполнения комплекса работ по заправке воздушных судов (ВС) гражданской авиации одним из ключевых вопросов является обеспечение безопасности полетов путем контроля качества авиатоплива непосредственно при выполнении заправочных операций. В настоящее время для обеспечения чистоты заправляемого авиатоплива на средствах заправки ВС устанавливаются фильтры-водоотделители с фильтроэлементами нормируемой степени очистки,

функционирование которых в рабочей зоне обеспечивает нормированные показатели очистки авиатоплива от воды и механических примесей. Как показывает практика, в процессе заправки ВС, по разным объективным и субъективным причинам, иногда возникают стохастические ситуации, при которых показатели качества выходят за пределы установленных нормативной документацией и не являются детерминированными, а последующее состояние такой системы описывается величинами, которые характеризуют за пределами низкий уровень очистки авиатоплива с негативными последствиями для безопасности полетов. В данной работе представлено математическое описание функционирования фильтров-водоотделителей в рабочей зоне, где обеспечиваются нормированные показатели качества авиатоплива в процессе заправки ВС. В статье рассмотрен вопрос блокировки заправки ВС в случае появления не нормированных технической документацией показателей качества авиатоплива, возникающих в силу ряда различных факторов, приводящих к негативным для безопасности полетов причинно-следственным связям. На основе математического описания предложен подход к созданию системы защиты и блокировки процесса заправки под рабочим наименованием «Барьер». Наибольший интерес для исследования представляют типовые фильтры-водоотделители, устанавливаемые на средствах заправки как оконечные технические устройства очистки топлива при заправках ВС.

**Ключевые слова:** фильтр-водоотделитель, фильтрующий элемент, средства очистки, нормированные и ненормированные показатели качества, рабочая зона, математическое поле, перепад давления, математическая модель, математическое обеспечение, метод наименьших квадратов, рабочее тело реактивных и турбореактивных двигателей воздушных судов.

## СПИСОК ЛИТЕРАТУРЫ

1. **Бордунов В.Д., Елисеев Б.П.** Стратегия правовой политики применения приложения 19 «Управление безопасностью полетов» // Научный Вестник МГТУ ГА. 2015. № 216. С. 5–10.
2. **Бурбаки Н.** Алгебра. Часть 2: Многочлены и поля. Упорядоченные группы: монография / Пер. с фр. В.Е. Говорова, Ю.И. Манина, А.В. Михалева и др., под ред. Ю.И. Манина. М.: Наука, 1965. 298 с.
3. **Браилко А.А.** Метод непрерывного мониторинга чистоты авиатоплива в технологической схеме топливообеспечения воздушных судов: дисс. ... канд. техн. наук. М.: МГТУ ГА, 2018. 134 с.
4. **Рыбаков К.В.** Авиационные фильтры для топлив, масел, гидравлических жидкостей и воздуха: учеб. пособие. М.: Машиностроение, 1982. 103 с.
5. **Виноградов В.Н., Гай Е.В., Работнов Н.С.** Аналитическая аппроксимация данных в ядерной и нейтронной физике. М.: Энергоатомиздат, 1987. 128 с.
6. **Лоран П.Ж.** Аппроксимация и оптимизация / Пер. с фр. Ю.С. Завьялова, Р.А. Звяжиной, Б.И. Квасовой, В.И. Шмырева, под ред. Г.Ш. Рубинштейн, Н.Н. Яненко. М.: Мир, 1975. 496 с.
7. **Магнус Я.Р., Катышев П.К., Пересецкий А.А.** Эконометрика. Начальный курс: учеб. пособие. 8-е изд., испр. М.: Дело, 2007. 504 с.
8. **Айвазян С.А.** Прикладная статистика. Основы эконометрики. Т. 2. М.: Юнити-Дана, 2001. 432 с.
9. Математическая энциклопедия. В 5 т. Т. 2: Д'Аламбера оператор – Кооперативная игра / Под ред. И.М. Виноградова. М.: Советская энциклопедия, 1979. 552 с.
10. **Митин И.В., Русаков В.С.** Анализ и обработка экспериментальных данных: учеб. пособие. 5-е изд. М.: МГУ, 2012. 44 с.
11. **Кельберт М.Я., Сухов Ю.М.** Вероятность и статистика в примерах и задачах. Т. 2: Марковские цепи как отправная точка теории случайных процессов и их приложения. М.: МЦНМО, 2010. 295 с.
12. **Марков А.А.** Распространение закона больших чисел на величины, зависящие друг от друга // Известия физико-математического общества при Казанском университете. 2-я серия. 1906. Т. 15. С. 135–156.

**13. Ландау Э.** Основы анализа. Действия над целыми, рациональными, иррациональными, комплексными числами. Дополнение к учебникам по дифференциальному и интегральному исчислению. 3-е изд. М.: КомКнига, 2010. 184 с.

**14. Бурбаки Н.** Очерки по истории математики. М.: Издательство иностранной литературы, 1963. 292 с.

**15. Bourbaki N.** Elements of the history of mathematics. Berlin: Springer, Heidelberg, 1994. 301 p. DOI: 10.1007/978-3-642-61693-8

**16. Дьёденноне Ж., Керрол Дж., Мамфорд Д.** Геометрическая теория инвариантов. М.: Мир, 1974. 278 с.

**17. Мюллер С.** Модернизация и ремонт ПК. 17-е изд. М.: Вильямс, 2007. 1504 с.

**18. Хульцебош Ю.** USB в электронике. 2-е изд., испр. СПб.: БХВ-Петербург, 2009. 224 с.

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