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## OPTIMIZATION OF AN AIRCRAFT FLIGHT TRAJECTORY IN THE GLONASS DYNAMIC ACCURACY FIELD

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The authors consider the problem of optimization of aircraft flight trajectories in air traffic management (ATM) on the basis of flexible routing technologies which involve the use of satellite navigation systems (SNS). It is shown that in optimizing a trajectory it is necessary to take into account the accuracy of track holding during the flight which depends on the accuracy of the navigation system and external flight path disturbances, e.g. wind. For solving the task of optimization the authors propose to use the theory of graphs. The technique of constructing a dynamic SNS accuracy field and representing it as a graph was developed. It is proposed that the SNS field be characterized by geometric dilution of precision changing both in space and in time. Based on the theory of graphs (A-star algorithm) the technique of constructing a trajectory of optimal length with changing the SNS accuracy and external flight path disturbances is proposed. The criterion of optimization based on minimizing the true track is offered. The cost function taking into account the track holding accuracy in navigating by SNS and effects of external flight disturbances is justified. The article presents the results of A-star algorithm application for constructing an optimal flight trajectory under conditions of SNS accuracy field variation and presence of prohibited zones in the provided airspace.

**Key words:** GLONASS, the geometrical dilution of precision, optimal trajectory, accuracy field, A-star algorithm, flexible routing.

### INTRODUCTION

The development of the global civil aviation is facilitated by the development and implementation of new technologies aimed at air traffic management system optimization. The present-day idea of air traffic management supposes that it must provide the efficient use of airspace and its high capacity for the conditions of highly saturated and intensive traffic at the same time maintaining and enhancing the flight safety levels. This feature of the ATM system is based on the airspace structure and use optimization, which also suppose the aircraft flight trajectory optimization. The optimization aimed at the trajectory distance shortening may result in savings in flight time, fuel consumption and also in reduction of environmental impact of aircraft.

The navigation technologies which provide enhanced efficiency of aircraft are the well-proven Area Navigation with upcoming Flexible routing and Trajectory Based Operations<sup>1</sup>. The above listed technologies imply the use of satellite navigation systems GPS and GLONASS as the main means of precise navigational positioning of the aircraft.

Flexible routing supposes the adequate level of situational awareness for both the flight crew and the ATCOs on the condition, the air traffic management in the airspace area, and also the recommended flight trajectory. At present, the level of situational awareness is determined, as a rule, by the meteorological information and forbidden areas in the airspace, and also the degree of coordinates precision and aircraft motion variables provided by navigation flight instruments (for instance, integrity control function for the satellite navigation system end user devices, the on-board complex navigation

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<sup>1</sup> Global Air Navigation Plan 2016-2030. International Civil Aviation Organization. Doc.9750-AN/963 [Electronic resource]. Montreal, Fifth edition, 2016. URL: [www.icao.int](http://www.icao.int). (accessed 16.05.2019).

systems which meet the ICAO<sup>2</sup> specifications [1]). These are the key factors for the crew decision making on the choice of the new trajectory, which will meet the certain conditions and provide the sufficient flight safety level.

However, the existing approach states that trajectory chosen by the crew or recommended by the ATCO does not take into consideration the possible performance alterations of the particular airspace, which may affect the trajectory precision. For example, the coordinate precision based on SNS data in different areas of airspace and at different moments in time may depend on the system geometrical dilution of precision. External flight path disturbances resulting from the wind changes, atmospheric turbulence may also change in time and space. Thus, the trajectory chosen at the entry to the airspace may not become optimal.

Accordingly, the optimum trajectory optimization solution should consider the changes in air navigational (provided by the airborne precise position instruments), air and meteorological conditions. So, the trajectory plotting must be based on its maintaining precision forecast within the alliterating airspace. This breeds the problem of the optimum flight path criterion, which would consider the forecast of trajectory maintaining precision.

A number of practical studies uses the criterion of minimum distance between the origin and the terminal point of the trajectory (the minimum length of desired track).at the same time the chosen trajectory must meet the flight safety conditions (avoid the forbidden areas, adverse weather conditions and prevent the near-collision risks).

It is worth mentioning that modern technologies of Air Traffic Management, for example, the area navigation, require the given degree of precision for the length of desired track, which is determined by the dispersion  $D_{TSE}$  of the flight management system total system error  $TSE^3$

$$D_{TSE} = D_{NSE} + D_{FTE} + D_{PDE}, \quad (1)$$

where  $D_{NSE}$  – Navigation System Error,  $D_{FTE}$  – Flight Technical Error,  $D_{PDE}$  – Path Definition Error.

The optimum path definition – the length of the desired track (LDT) is characterized as  $L_{LDP}$ . Due to the possible deviations from the desired track the aircraft will follow the track made good (TMG), which is characterized by the length  $L_{TMG}$ . Here  $L_{TMG} = L_{LDP} + \Delta L_{TSE}$ , where  $\Delta L_{TSE}$  is the increment addition to the length of desired path due to deviations.

Let us consider two possible flight trajectory definitions (figure 1): optimal, which features the minimum desired path length  $L_{LDP1}$ , and non-optimal, which is characterized by the length  $L_{LDP2} > L_{LDP1}$ .



Fig. 1. Desired track for various trajectory holding errors, here Л3П1 – LDP1; Л3П2 – LDP2; ЛФП1 – TMG1; ЛФП2 – TMG2

<sup>2</sup> Performance based navigation guidance Doc.9613-AN/937 4<sup>th</sup> edition The International Civil Aviation Organization Montreal, 2013. Available at: [www.icao.int](http://www.icao.int) (accessed 16.05.2019).

<sup>3</sup> The same source.

At certain conditions which depend on the trajectory precision (at  $\Delta L_{TSE1} > \Delta L_{TSE2}$ ), the situation is possible when  $L_{LDP2} + \Delta L_{TSE2} < L_{LDP1} + \Delta L_{TSE1}$ . Then, the inequation is fair,  $L_{TMG2} < L_{TMG1}$   $L_{TMG2} < L_{TMG1}$  i.e. the length TMG when flying on the non-optimal LDP will be less than in case of flying on the optimal estimated trajectory. Thus, the choice of the new trajectory must depend on the precision of maintaining one during further flight.

The solution for the problem of flexible routing may be implemented in two stages. The first one is the optimal trajectory plotting for the given area of the airspace. The second stage supposes following the optimal flight trajectory with minimum deviation from it.

In order to build the optimum flight trajectory artificial intelligence methods and algorithms may be applied [2]. For instance, paper [3] is focused on the task of plotting flexible approach trajectories using genetic algorithms. Research works [4, 5] use the methods and algorithms of the graph theory. In particular, paper [5] studies the algorithm constructed considering the weather conditions, which allows to decrease the flight time and fuel consumption.

In order to solve the problem of optimum trajectory pilotage, the methods of optimal control theory are widely used. Here, a variety of optimization criteria are applied. For instance, paper [6] studies the multicriteria trajectory optimization problem, and paper [8] studies the time-referenced optimal trajectory 4 D trajectory.

However, the mentioned above research papers and the others study the trajectory optimization and control disregarding the problem of trajectory precise maintaining considering the changes of aerona-vigational situation and air picture within the given airspace area.

Modern and advanced navigation technologies are based on the use of satellite navigation, as the most precise position finding aid. So, as a pilotage sensor, which determines the navigation system disperse error value  $D_{NSE}$  in the expression (1), the satellite system receiver will be taken.

The satellite navigation system positioning precision depends on the navigation satellite position relative to the aircraft and is determined by the geometrical dilution of precision, which changes both in time and space [1, 9]. Accordingly, the SNS positioning is altered as well, which leads to the trajectory maintaining precision change. So, the optimum track made good length when SNS is being used must be plotted considering its forecast maintaining precision in the changing space and time SNS precision field, featuring the values of the geometrical dilution of precision.

The present paper is aimed at solving the tasks of optimum trajectory plotting using the graph theory methods (A-star algorithm). The difference from the popular solutions is in the use of the new optimization criterion – the minimum track made good length, which considers the trajectory maintaining precision. To solve this task it is necessary to establish relation between the track made good length with the SNS precision field properties in the given airspace area and outer flight path disturbances; to develop the methods of SNS precision field graph presentation; to investigate the algorithm effectiveness and efficiency in different conditions, including the forbidden airspace areas.

## RESEACH METHODS

In the context of aviation traffic system, the airspace and its elements, between which the aircraft are travelling may be presented as a network of routes. Such networks may be modelled as graphs, graph nodes connected together in a certain manner allow to plot the optimal flight route between the point of entry into the airspace area and the point of exit from it [10]. The most common shortest-path algorithms for the graph theory are the Dijkstra algorithm and  $A^*$  (*A-star*) [4].

Dijkstra algorithm finds the shortest trajectories from the given graph node to all the nodes remaining. As this takes place, every node gets its weight, which characterizes the distance to it from the neighbor node (edge weight). The optimal is the trajectory from the initial node to terminal node for which the edge weight sum ( $F(L_{LDP})$ , where  $L_{LDP}$  – desired path length) is minimum.

The A-star algorithm uses the informed search strategy, thus combining the mathematical and heuristic approaches. The heuristic approach supposes the use of specific for the problem area knowledge, which allows to apply the A-star algorithm in the artificial intelligence systems and thus to reduce the computational costs. For the task of plotting the optimal desired path length the heuristic approach is based on the fact, that the shortest distance between two points en route in navigation is the partial orthodromic route.

The A-star algorithm minimizes the cost function  $F(L_{LDP}) = G_{0ij} + H_{ij}$ . Here  $G_{0ij}$  – the cost function of reaching node  $(i,j)$  from the initial graph node (the point of aircraft entry to the airspace),  $H_{ij}$  – the heuristic estimate from node  $(i,j)$  to the terminal graph node (the point of leaving the airspace). For this problem the heuristic estimate of the distance is the length of the partial orthodromic route.

It is supposed to take into consideration in cost computing the TSE, presenting the function as

$$F(L_{TMG}) = G_{0ij}(1 + K_{ij}(TSE)) + H_{ij}, \quad (2)$$

where  $G_{0ij}$  – the length of desired path from the terminal node to the graph node  $(i,j)$ ;  $K_{ij}(TSE)$  – coefficient which considers the desired path length extension due to total system error  $H_{ij}$  – heuristic estimate of the distance to the node in question to the terminal node (the length of the orthodromic path).

The optimal flight trajectory must meet the condition

$$F(L_{opt}) = \sum_{i=1, j=1}^{e(G)} F(L_{TMGi,j}) \rightarrow \min, \quad (3)$$

where  $L_{opt}$  – the length of track made good along the optimal trajectory being plotted;  $e(G)$  – the number of graph edges used to plot the trajectory from the initial to the terminal graph node,  $F_{TMGi,j}$  – the weight of the graph node number  $(i,j)$ .

The only limitation for optimal trajectory plotting is the possibility of having forbidden areas in the given airspace area.

Considering the error *NSE*, *FTE* and *PDE* independence, the expression is

$$K_{ij}(TSE) = K_{ij}(NSE) + K_{ij}(FTE) + K_{ij}(PDE).$$

The coefficient value  $K_{ij}(PDE)$  depends on the precision of flight path computation from the flight guidance computer. Thereafter let us suppose that this error is much less than the other components of TSE and not consider it any more.

The coefficient value  $K_{ij}(NSE)$  is determined by the precision of the airborne navigation system. When using SNS the aircraft positioning precision will be determined by the SNS airborne receiver precision. This statement is also fair for using it to correct the coordinates determined by the airborne reference system, for example the inertial navigation system. Here, the precision of the SNS receiver is determined by the error in the pseudo distance measuring to satellites and the value of the position dilution of precision in the point of observation.

The value of coefficient  $K_{ij}(FTE)$  depends on the aircraft aerodynamic performance and its weight, and also the airway conditions (wind, atmospheric turbulence etc).

Let us suppose that the desired flight path length increment due to navigational system error may be approximated to

$$\frac{d}{dt} \Delta L_{NSE}(t) = -\alpha_{NSE} \Delta L_{NSE}(t) + \sigma_{NSE} \sqrt{2\alpha_{NSE}} w_{NSE}(t), \quad \Delta L_{NSE}(t_0) = 0, \quad (4)$$

where  $\alpha_{NSE}$  – constant time error correlation;  $\sigma_{NSE}$  – standard uncertainty;  $w_{NSE}$  – forming white Gaussian noise with zero mathematical expectations and one intensity.

Moreover, the desired flight path increment at SNS navigation is the product  $\sigma_{NSE} = PDOP \cdot \sigma_R$ , where  $\sigma_R$  – standard uncertainty of pseudo distance measuring to satellites.

According to the data obtained from the real-time monitoring of 16.05.2019, from the website of Russian system of differential correction and monitoring<sup>4</sup> the maximum pseudo distance measuring error to GLONASS satellites in the point of observation was 15.74 m. Considering the possible anomalies in the receiver operation and its installation on a highly dynamic object, let us assume the value  $\sigma_R = 50$  m.

As the typical value for the constant time error correlation of the SNS receiver let us choose  $\alpha_{NSE} = 0.01$  Hz, supposing that using the measurement antialiasing procedures the outgoing error of SNS receiver is a sufficiently narrow-band (slow moving) process.

To find the value  $K_{i,j}(NSE)$  empirically, the Monte-Carlo method was applied. As a result it was determined that at the chosen initial data the values of  $K_{i,j}(NSE)$  lie within the range 0.001–0.025 with PDOP ranging from 1 to 6.5. At  $\alpha_{NSE} = 0.1$  Hz and PDOP changing from 1 to 6.5 values  $K_{i,j}(NSE)$  are within the range 0.03–0.17. Thus, the error fluctuation spectrum spreading at SNS receiver output leads to track made good length increment, which proves the adequacy of the model used.

To find the value of it is possible to use the well-known model of controlled flight at external trajectorial effects [11]

$$\begin{aligned} \frac{d}{dt} \Delta L_{FTE}(t) &= \Delta W(t), \quad \Delta L_{FTE}(t_0) = 0, \\ \frac{d}{dt} \Delta W(t) &= a(t), \quad \Delta W(t_0) = 0, \\ \frac{d}{dt} a(t) &= -\delta \cdot a(t) - \beta \cdot \Delta W(t) + \sqrt{2\delta\sigma_a^2} \cdot n_a(t), \quad a(t_0) = 0, \\ \frac{d}{dt} W_0(t) &= 0, \quad W_0(t_0) = W_{00}, \end{aligned} \tag{5}$$

where  $\Delta L_{FTE}(t)$  – desired flight path length increment due to trajectorial distortions;  $\Delta W(t)$  and  $a(t)$  – vector projection of aircraft on desired path ground speed and acceleration fluctuations;  $\delta$  and  $\beta$  – coefficients characterizing the spectral density of accidental changes in acceleration due to external effects, object type and the terms of motion;  $\sigma_a^2$  – acceleration fluctuation dispersion;  $n_a(t)$  – forming white Gaussian noise with the zero mathematical expectations and one intensity;  $W_0$  – rated speed.

Coefficients  $\delta$ ,  $\beta$  and parameter  $\sigma_a^2$  may be calculated as  $\delta = b + v$ ,  $\beta = bv$ ,  $\sigma_a^2 = v^2 b \sigma_u^2 / \delta$ ,  $b = V/L$ ,  $V$ , where  $b = V/L$ ,  $V$  – aircraft air speed,  $L = 200...1000$  m atmospheric turbulence scale,  $v = 0.1...0.01$  c<sup>-1</sup> parameter, dependent on the aircraft type and flight conditions,  $\sigma_u = 0.4...2.7$  m/s wind speed fluctuations standard uncertainty.

Using the Monte-Carlo method it was found that at typical values  $\delta = 0.34$  Hz,  $\beta = 0.0044$  s<sup>2</sup>,  $\sigma_a = 0.1...0.5$  m/s<sup>2</sup> the coefficient  $K_{i,j}(FTE)$  values are within the range 0.003–0.01.

<sup>4</sup> Russian system of differential correction and monitoring. Available at: <http://www.sdcm.ru> (accessed 16.05.2019).

The particularity of the task of plotting the optimal track made good length is that during the flight with many factors acting the errors NSE and FTE may alter. This effect is supposed to be considered while forming the weights for graph nodes in accordance with expression (2).

Let us consider the method of graph forming for the case when the node weights are determined only with NSE. For this purpose, we need to build the precision field GLONASS in the given airspace area considering the forecast of its alterations during the flight.

The GLONASS precision field characteristic feature is going to be the airspace distribution of PDOP points. The aggregate of points where PDOP value is the same or within the given limits will allow to build the precision field standard uncertainty as the domains of common values PDOP [10]. The precision field forecast for any moment of time and any point of airspace is possible, as PDOP is a changing, yet determined characteristics of standard uncertainty precision. At this being stated, PDOP depends on the current position of the aircraft and NS constellation, which may be computed for any moment of time at any point of airspace using the data from SNS almanac.

### SOFTWARE IMPLEMENTATION

To carry out the research in the graphical programming environment software complex was developed by *LabVIEW9* (figure 2). Its structure is determined by the following special aspects:

- the trajectory optimization task is solved in the geodetic system of coordinates (latitude  $B$ , longitude  $L$ , height  $H$  over the terrestrial ellipsoid surface);
- the motion of NS on the orbits is given in the earth-fixed geocentric system  $OXYZ$ ;
- to build a graph, the points of the airspace, where PDOP can be determined are projected using the Gauss-Kruger projection onto the surface plane. At the same time the equality of distances between the neighbor graph nodes is provided.



Fig. 2. Flow chart of the software complex

The software complex comprises: ВПр – the virtual device, which provides the user-friendly interface, 1– the module of graph node formation on the surface  $\Gamma(x, y)$ , 2 – the module of graph geodetic coupling  $\Gamma(B, L)$  to the geodetic coordinate system; 3 – the module of GLONASS almanac transformation; 4 – the module of orbital motion computation; 5 – the module of computation of PDOP values at the graph nodes; 6 – the module of trajectory plotting; 7 – the module of initial and terminal condition computation for the plotted trajectory; 8 – the module of optimal trajectory choice; Координаты начальной и конечной точек траектории, границ запретных зон – coordinates of initial and terminal points of the trajectory, forbidden area boundaries; Координаты границ

зоны ВП – airspace area boundary coordinates; Эфемериды НС – NS ephemeris; Координаты вершин графа, принадлежащих оптимальной траектории – Coordinates of graph nodes which belong to the optimal trajectory.

Module 1 forms the coordinate grid (graph node) Gauss-Kruger projection in the airspace with a fixed pitch. The value PDOP is calculated at the graph nodes. Module 2 is intended to be used for graph node coupling to the Earth surface considering its curvature.

Module 3 transforms data assets input onto the virtual device from the GLONASS almanac (ephemeris) as vectors  $\overline{\text{Lam}}$  – longitude of ascending node;  $\overline{\text{dI}}$  – orbit inclination angle correction;  $\overline{\text{w}}$  – perigee argument;  $\overline{\text{e}}$  – ellipticity;  $\overline{\text{dT}}$  – a correction to average satellite nodical period;  $\overline{\text{dT}^2}$  – orbit time rate;  $\overline{\text{T}w}$  – time of passing the first node) each containing 24 elements (the number of NS in the orbit group). The ephemerid data in such a way are convenient for further transformations in the program module.

Module 4 forms three vectors  $(X_i, Y_i, Z_i)$  which contain the corresponding coordinates of NS in the rectangular geodetic coordinate system for the moment  $i$ . Module 5 determines NS in sight, and computes PDOP values at the graph nodes (forms the weighed graph). Module 6 finds the optimal track in a weighed graph considering the forbidden airspace area limits.

Module 7 sets the airspace area dimensions, the position and the size of forbidden areas within the airspace and also the trajectory parameters (speed, height, heading).

Using the virtual device, the GLONASS almanac is uploaded from the website<sup>5</sup> the aircraft travel parameters are given, the airspace area boundaries are introduced, and trajectory limitations are shown. The virtual device faceplate displays the data out (optimal trajectory, estimated track made good length etc.).

## THE EXPERIMENT RESULTS

The trajectory optimization may be implemented in a static (built for a certain moment of time) precision field GLONASS. However, the experiment results [10] show that the structure and the value of PDOP for static precision fields at different moments of time varies.

Thus, the trajectory, which is optimal in the static field may be non-optimal due to its alterations en route. In these conditions only the computation of dynamic field will allow to obtain information on the actual precision of GLONASS en route coupled to the time, and to plot the optimal 4 D trajectory.

The dynamic field is built based upon the forecast values of PDOP along the plotted trajectory for the estimated moments of time when the aircraft is en route.

Figure 3, shows the dynamic field built on the estimated PDOP values at the moments of time the aircraft travels along the given route. The field areas correspond to the different ranges of PDOP changes within their boundaries. Basing on the GLONASS dynamic field of precision obtained the graph is being built (figure 3 b) the nodes of which have weights, dependent on PDOP values distribution in time and within the airspace area chosen.

<sup>5</sup> Information and analysis center for positioning, navigation and timing. Available at: <https://www.glonass-iac.ru/> (accessed 16.05.2019).

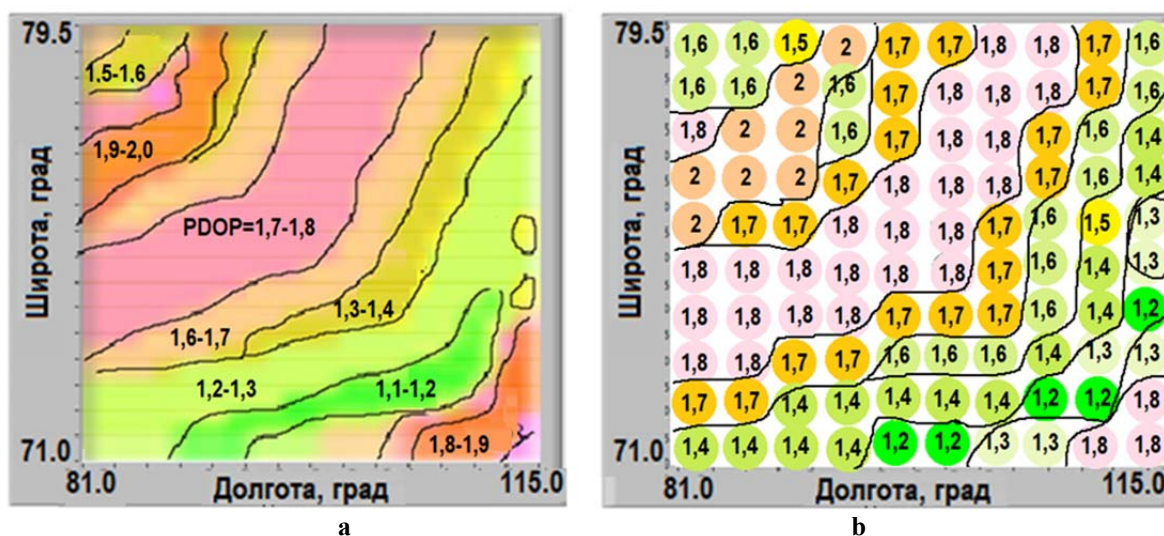


Fig. 3. The representation of the GLONASS field of precision as a graph, here Широта, град – Latitude, degrees; Долгота, град – Longitude, degrees

The results of the experiments have shown that the dynamic precision field is less homogenous, than the static one, besides, the range of possible PDOP value variants is wider compared to the static field.

To solve the task of optimal trajectory plotting the A-star algorithm was applied. At the same time its effectiveness and efficiency in different conditions, characterizing the airspace (forbidden areas if any) and GLONASS precision field (minor or major PDOP alterations) were subjects to survey.

As the results obtained from modelling have shown, at rather precise GLONASS field and its minor variations ( $PDOP < 2.0$ ) the optimal trajectory gets in line with the orthodromic path. At the same time the length of the track made good may change within 2–3% depending on the mean value  $PDOP_{av}$  during the flight en route. By choosing the time of joining the airway to meet the minimum  $PDOP_{av}$ , it is possible to achieve the reduced track made good length.

Figure 4 shows the optimal routes plotted in significantly non-homogeneous precision field ( $PDOP = 1.1 \dots 6$ ). The routes are plotted for two different moments of aircraft entry to the initial point of the route (P.A).

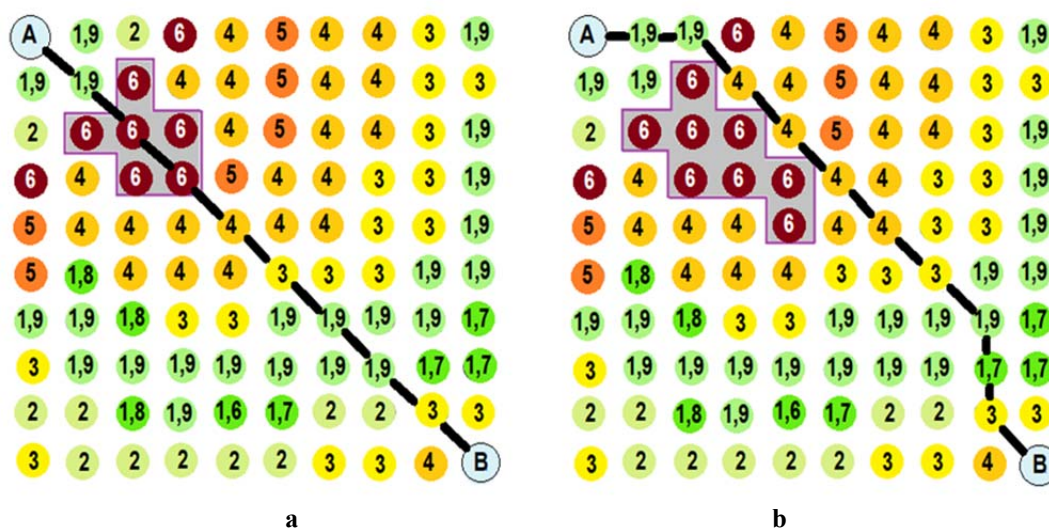


Fig. 4. Optimal routes with significant variations  $PDOP$



The results obtained show that in case of minor sizes of PDOP “bad” values the optimal route may be in a line with the orthodromic path (figure 4 b). In this example the orthodromic track made good is 1416ю3 km, and the optimal trajectory is 1399.5, which is almost 17 km (1.2) less.

Let us view the problem of optimal trajectory plotting for the case of forbidden areas in the airspace. Figure 5 a and b show the routes, plotted in the GLONASS precision field at minor alterations in PDOP values ( $1.2 < PDOP < 1.9$ ), and at minor range of variations ( $1.3 < PDOP < 10$ ) (figure 6 a and b) at different configuration of forbidden areas.

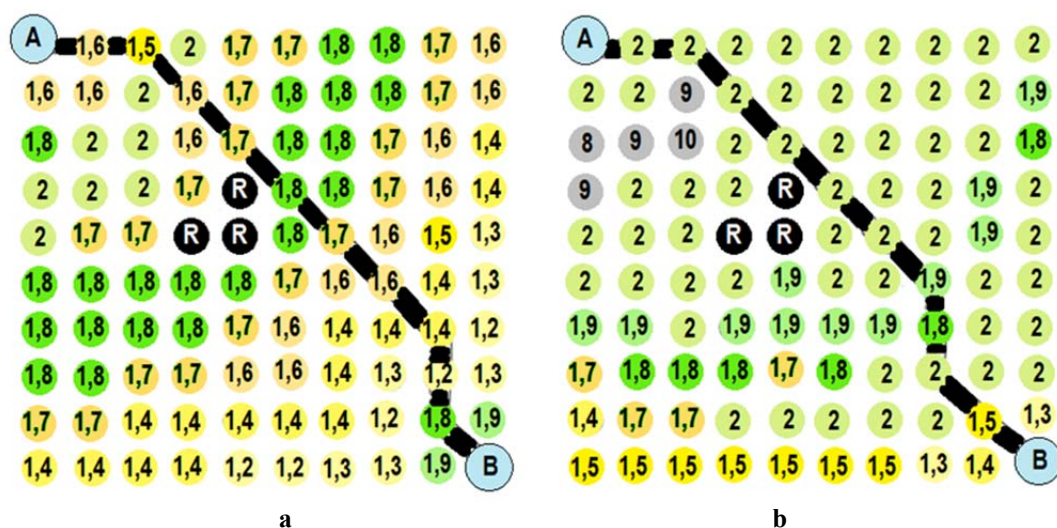


Fig. 5. Optimal routes in the presence of restricted areas and minor variations of *PDOP*

The research carried out has shown that the result of trajectory optimization depends both on the GLONASS precision field, and also on the size and configuration of forbidden areas in the airspace area in question.

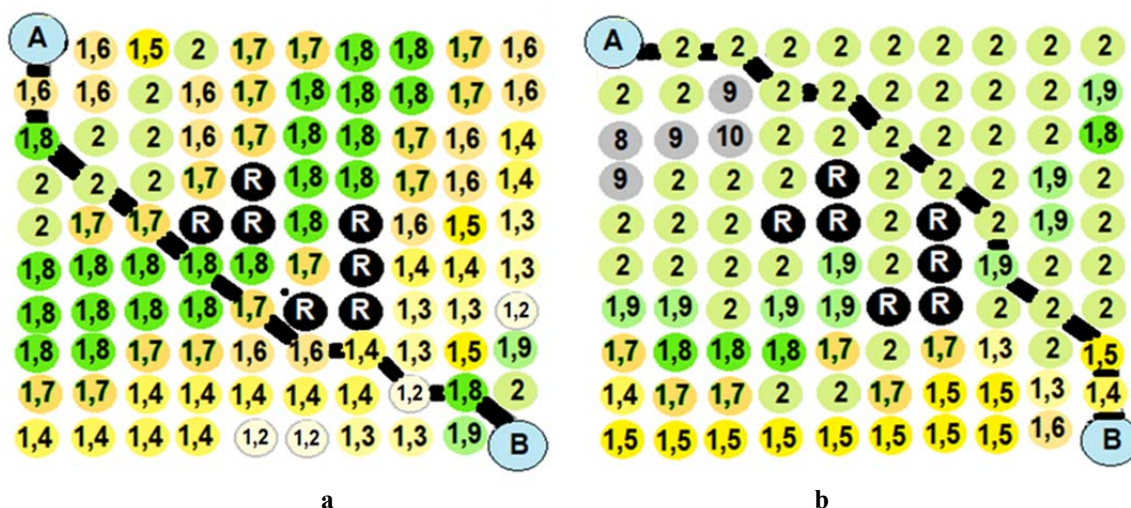


Fig. 6. Optimal routes in the presence of restricted areas and significant variations of *PDOP*

Figure 7 shows the trajectories of aircraft flight for two cases: the heuristic forbidden area avoiding trajectory (Trajectory 1) and the optimal trajectory (Trajectory 2).

The results obtained from modelling show that the track made good at optimal trajectory is 1392.9 km, and heuristic trajectory – 1451.3 km, which is almost 58 km (4%) longer, than the optimal trajectory flight.

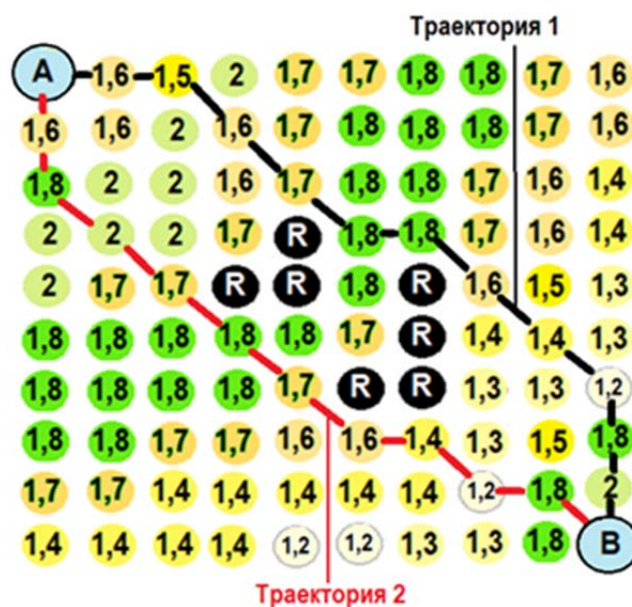


Fig. 7. Optimal and heuristic flight paths,  
here Траектория 1 – Trajectory 1; Траектория 2 – Trajectory 2

## CONCLUSION

The results obtained prove the effectiveness of the graph algorithms application for plotting the optimal track made good at different characteristics of the GLONASS dynamic precision field with the forbidden areas.

The suggested new approach to the optimal trajectory plotting allows to consider the precision of trajectory maintaining using GLONASS to navigate, and also the presence of forbidden areas in the airspace area given. This may be applied at preliminary flight planning and during the flight itself having the software installed onto the airborne computer. The optimal trajectory data available will improve the situational awareness at optimal routing decision making in conditions of changing airspace situation and the navigational instrument precision.

If the suggested criterion of optimal trajectory choice is being applied, the reduced track made good is provided. In addition, the track made good length saved depends on the GLONASS precision field properties and is rather significant for long -haul flights. For instance, for a route which covers 5000 km, the track made good length is reduced by 60 km, which equals to a time saving of 5 minutes. Even insufficient flight time saving in the circumstances of air traffic of high intensity will provide the airlines with significant economy levels.

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#### АЛГОРИТМЫ ОПТИМИЗАЦИИ ТРАЕКТОРИЙ ВОЗДУШНЫХ СУДОВ ПРИ ГИБКОЙ МАРШРУТИЗАЦИИ

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Рассмотрена задача оптимизации траекторий полета воздушных судов при организации воздушного движения на основе технологий гибкой маршрутизации, предполагающих использование спутниковых навигационных систем (СНС). Показано, что при оптимизации траектории необходимо учитывать точность ее выдерживания в процессе полета, которая

зависит от точности навигационной системы и внешних траекторных возмущений, например ветра. Для решения задачи оптимизации предложено использовать методы теории графов. Разработана методика построения динамического поля точности СНС и его представления в виде графа. Предложено поле СНС характеризовать значениями геометрического фактора, изменяющегося как в пространстве, так и во времени. На основе теории графов (алгоритм *A-star*) предложена методика построения оптимальной по протяженности траектории при изменении точности СНС и внешних траекторных воздействиях. Предложен критерий оптимизации, основанный на минимизации длины линии фактического пути. Обоснована функция стоимости, учитывающая точность выдерживания траектории при навигации по СНС и влияние внешних траекторных возмущений. Представлены результаты применения алгоритма *A-star* для построения оптимальных траекторий полета в условиях вариаций поля точности СНС и наличия запретных зон в предоставляемой зоне воздушного пространства.

**Ключевые слова:** ГЛОНАСС, геометрический фактор, оптимальная траектория, поле точности, алгоритм *A-star*, гибкая маршрутизация.

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