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MODELING OF OIL POLLUTION OF ARTIC SEA COASTAL AREAS

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This article studies the elastic filtration oil drive of oil in a layer based on the estimation of risks of environmental oil pollution because of accidental releases. The model of oil spillage and resorption by the precoat is based on continuity equation and Darcy rule as well as on equations of state taking into account fluid compressibility due to pressure. Filtering area is a line between the precoat and air. Oil filtering area is limited by soil surface below and by free surface above, its equation is known beforehand and is to be defined. The case of soil pollution from the point source, which is the point of fracture of pipeline or borehole, is considered. Upper and approximate estimates of the oil pollution radius due to different types of underlying terrains and to oil characteristics as well as to environmental conditions. The dynamics of oil free surface depending on spillage radii is calculated and presented. The estimates of temporary duration of oil filtering by the precoat in terms of light ends and soil type are made. The thickness of the oil film and the square of the spill upon condition of constant speed of oil spillage, horizontal position of underlying terrain and the proximity of pressure to normal are determined. For the numerical implementation of the model different cases of oil spillage were considered. Under given values of air temperature, soil porosity and filtration speed the pollution radii according to light end, the time from the moment of accident till the leakage suppression and the speed of oil spillage was calculated.

Key words: model of spreading and absorption of oil by soils, elastic filtration mode, filtration area, soil porosity, radius and square of pollution, oil fractions.

INTRODUCTION

In the coastal zone of the Russian Arctic seas there are many oil deposits, a great number of operating there transport infrastructure objects, the number of which, due to the increased production volumes, is planned to increase significantly in the nearest future.

Exploitation of oil fields as well as of a transport system is accompanied by significant amounts of oil emissions in the environment, especially during the accidents. This causes great damage to nature, aggravated by long terms of recovery of the Arctic ecosystems and high costs of the impact elimination.

Therefore, for designing such objects we need to estimate the scale of potential pollution so that on the basis of the obtained estimates we could develop effective measures to eliminate the consequences of accidental spills. On the other hand, a comparison of possible consequences of various alternatives for the development of the deposits and especially for the objects of transport infrastructure allows us to choose more efficient version of their placement in order to minimize the effects of oil spills.

RESEARCH METHODS AND METHODOLOGY

You can use two approaches for obtaining estimates of spatial and temporal scales of pollution: a method of analogy and modeling of processes of spreading and transformation of petroleum hydrocarbons in the environment. The first approach is limited by the nature itself, as it rarely repeats, moreover, the accidents are also very diverse. The second approach seems to be more promising to us, as it allows us to simulate the whole process of oil pollution evolution for various scenarios of accidents.

A mathematical model of oil flowing on the surface of the earth should take into account the following processes and circumstances:

• physico-chemical properties of oil, which the value of light fractions evaporation and viscosity coefficient depend on;

• the slope of the surface, which affects the rate of spreading, and thus, the size and shape of the area of pollution;

• the properties of the underlying surface - the nature of vegetation, soil porosity and the water cut in the soil, which the rate of spreading and the value of the absorption of petroleum hydrocarbons by soils depend on.

It is natural that the oil pollution processes are influenced by other factors, such as air temperature, which the rate of evaporation depends on, and they should also be taken into account in the models, but the factors listed above are basic.

One can take into account the spreading of oil and its absorption by soils using the continuity equation and Darcy's law [1, 2]:

$$\frac{\partial(\rho m)}{\partial t}+div(\rho\vec{v})=\rho Q,\quad \vec{v}=-k\ grad(h), \tag{1}$$

where $h=p/\rho g+z$ – the hydraulic head or the piezometric head, ρ – the density of the liquid, m – the porosity of the soil, Q – the intensity of sources, k – the filtration coefficient. Further, as accepted, t – time, $\vec{v}$ – filtration velocity, p – pressure, g – gravitational acceleration, z – vertical coordinate.

System (1) is consistent with the equations, which take into account medium compressibility depending on the pressure (equations of state):

$$\rho=\rho_0\left[1+\beta_l\left(p-p_0\right)\right],\quad m=m_0+\beta_s\left(p-p_0\right),$$

where ρ_0 and m_0 – density and porosity at pressure p_0. β_l and β_s respectively are called the coefficients of oil and soil compressibility. Moreover, it is assumed that the pressure in the medium does not differ much from the normal pressure p_0. Therefore, system (1) is classically linearized and reduced to a special kind of inhomogeneous heat equation, called the elastic mode, or the piezoconductivity equation [3]:

$$\frac{1}{a}\frac{\partial h}{\partial t}-\Delta h=\frac{Q}{k},\quad a=\frac{(1-p_0\beta_l)k}{(\beta_s+m_0\beta_l)\rho_0 g}. \tag{2}$$

a is called the piezoconductivity coefficient, which characterizes the rate of redistribution of the pressure in the elastic layer and has the dimension m^2/s. First, we assume that there is no filtration and the rate of oil absorption (ε) by the soil surface $(z=0)$ is known. Hence, we obtain boundary conditions:

$$h\Big|_{t=0}=0,\quad k\frac{\partial h}{\partial z}\Big|_{z=0}=\varepsilon,\quad t>0. \tag{3}$$

Filtration area is bordered by the soil surface, as well as by the free surface, which is not known a priori, has to be defined and is the border between the soils, filled with liquid, and air. Therefore, the pressure along the filtering area equals the atmospheric one $(p=0)$. In this problem in a place of an accident there is located a point source with cylindrical coordinates $(r=0,\ z=H)$ consequently:

$$Q=\frac{q(t)}{2\pi\cdot r}\delta(r)\delta(z-H). \tag{4}$$

Here $q(t)$ – volume power of source, δ – Dirac delta function [4]. Solving the problem (2), (3), (4), using Duhamel's principle, we find the potential of rates in the form:

$$-kh = \frac{\varepsilon \cdot z}{\sqrt{\pi}} \int_{\frac{z}{\sqrt{4at}}}^{+\infty} e^{-\xi^2} \frac{d\xi}{\xi^2} - aq(t) * G_H. \qquad (5)$$

Symbol $*$ means the convolution of the originals (in the Laplace transform). Influence function:

$$G_H = \frac{e^{-\frac{r^2+(H+z)^2}{4at}} + e^{-\frac{r^2+(H-z)^2}{4at}}}{(\pi 4at)^{3/2}}.$$

Formula (5) has a rather general form and responds to various modes of the leakage spreading. One gets a simple and interesting solution if $q(t) \equiv q_0$, when $0 < t < T$, and zero otherwise. It corresponds to the reality quite accurately, where T – the moment of leakage elimination, q_0 – the production rate (the averaged volume of oil flowing from the source per unit of time). Thus, all the characteristics of the spreading of fluid on the soil surface can be found analytically. Namely, assuming $h = z$, we find the implicit free surface equation, and then the area of pollution, as the intersection of this surface and the soil surface $z = 0$. On this basis, we obtain the law of the evolution of the radius of pollution until the moment T:

$$r^2 = -H^2 + 4at \cdot \ln \frac{q_0}{\pi\varepsilon 4at}. \qquad (6)$$

The choice of the model imposes the corresponding restrictions: for some values of the parameter t the right side of (6) must be positive, as the decreasing radius of pollution is a consequence of leak elimination. This is expressed by the estimates: $q_0 \geq e\pi\varepsilon H^2, q_0 \geq e\pi\varepsilon 4aT$. Then $(t \geq T)$, the dynamics of the pollution radius is described by the equation:

$$\frac{4\varepsilon\sqrt{\pi at}}{q_0} = \frac{erf\left(\frac{\sqrt{r^2+H^2}}{\sqrt{4a(t-T)}}\right) - erf\left(\frac{\sqrt{r^2+H^2}}{\sqrt{4at}}\right)}{\sqrt{r^2+H^2}}, \qquad (7)$$

which gives us the upper limitation on the radius of pollution and duration of the process:

$$r \leq \sqrt{0.0678\, q_0/\varepsilon - H^2}, \quad t \leq \frac{q_0}{\pi\varepsilon 4a}. \qquad (8)$$

q_0 and ε are found by means of the easily defined volume of oil, flowed out from the source V_0 and filtered by the soil $V_1 = \int_0^T \int v_n ds$ during time T:

$$V_0 = q_0 T, \quad V_1 = \pi\varepsilon a T^2 \left\{1 - \frac{H^2}{aT} - 2\ln\left(\frac{\pi\varepsilon 4aT}{q_0}\right)\right\}.$$

So, the method of estimating consists in calculating of hypotenuse $d = 0.0678\,q_0/\varepsilon$ in a right triangle with legs equal to the height H and the desired radius of pollution r (fig. 1).

This solution imposes quite strict restrictions on the scenarios of accidental spills that limits the applicability of the model, but, in its turn, makes it easy to evaluate the radius of the pollution area, without large computational costs.

Let us illustrate the method on the example of the following scenario: as a result of an accident at the well there occurred an oil spill, leak detection and repairs took three days. The well gives light fraction oil. It is known that during this time 250 t of oil flowed out, 50 t of which have been absorbed by the soil. Accidental spill occured in summer, at a temperature of 15 °C and atmospheric pressure, in a tundra region with peat soils.

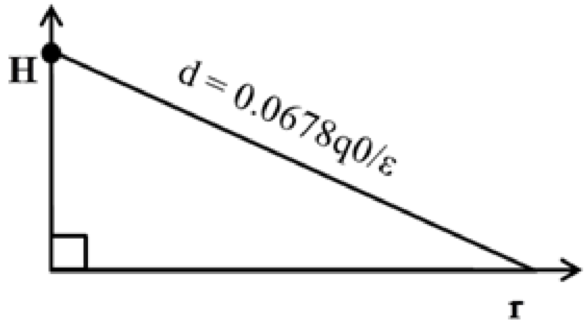


Fig. 1. Method of evaluation of the radius of pollution (• - source)

Peat has high porosity, up to 90–95 %, decreasing with the increase in the degree of decomposition, as well as a wide range of variation of the compressibility coefficient, from 1.5 to 80 MPa^{-1}, reaching the upper limit in heavily watered, poorly decomposed peats [5]. On this basis, we take the porosity of peat equal to 0.92 and compressibility coefficient – 75 MPa^{-1}. The range of peat permeability coefficient variation has been calculated by using the filtration coefficient values [1, 6], and for calculations the value equal to 800 D was taken. Such physical properties of crude oil as density and viscosity, are taken from [7]. Compressibility coefficient, in its turn, is calculated using the empirical formula [8].

Eventually, we have the following parameters for calculation: $H = 0.01$ m, $T = 72$ h, $V_0 = 304.878$ m^3, $V_1 = 60.975$ m^3, $\beta_s = 75$ MPa^{-1}, $\beta_l = 0.822$ GPa^{-1}, $k_0 = 800$ D, $m_0 = 0.92$, $\rho_0 = 820$ kg/m^3, $\mu = 8$ cSt, where μ – kinematic viscosity, required to determine the filtration coefficient $k = \frac{m_0 g}{\mu}$.

The upper estimates for the time and radius of pollution, calculated according to the formulas (8), are as follows: $t \le 398$ h, $r \le 44.24$ m. Approximate estimates, calculated on the basis of the main equation (7): $t \approx 151$ h, $r \approx 38.93$ m. Based on the assumption of horizontality of the soil surface, the area of pollution is bounded by a circle of radius r, that allows us to calculate its square (S). Then $S \approx 4761$ m^2, $S \le 6149$ m^2. Fig. 2 shows the dynamics of the free surface over time. Reflection symmetry with respect to z allows us to display only the cross section with the plane $\varphi = 0$.

In order to verify the model, there was written a program, that carries out the calculation of oil spills model scenarios.

Development of the program was carried out with the help of OpenSourse tools such as: Qt 4.8.5 and Qwt 6.1.0, distributed under open source license, such as GNU LGPLv2.1 or its equivalents.

With the obtained analytical solution, we create a simple scenarios modeling algorithm:

1) Start;

2) Input of the initial data;

3) Transition of all the measurement units in the SI system;
4) Verification of the initial data, and if yes, to step 5, otherwise to step 1;
5) If there is selected the input of V_0 and V_1, then make the computation of q_0 and ε;

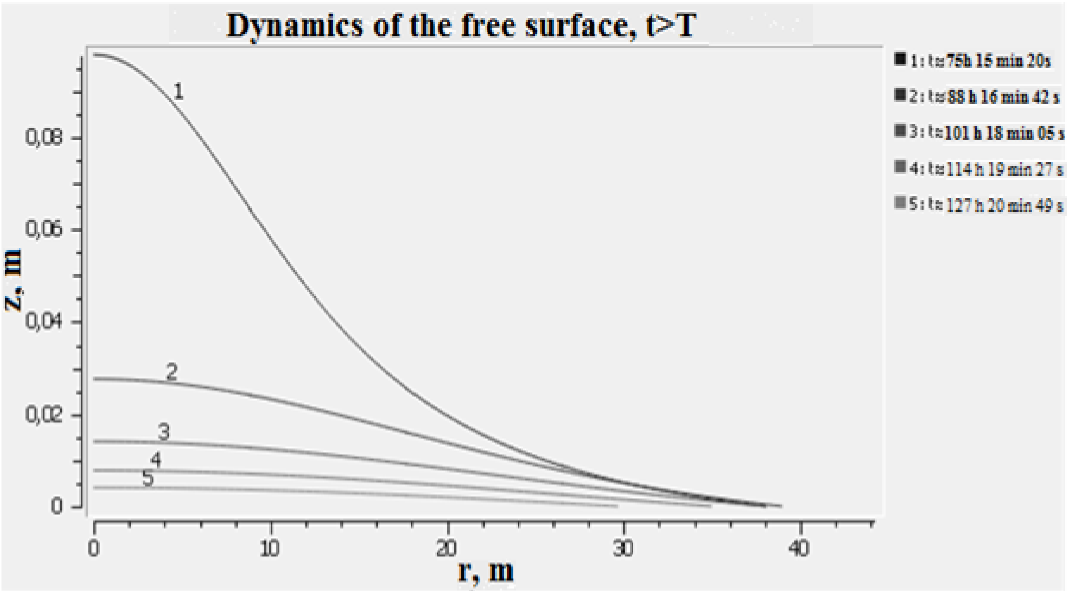


Fig. 2. Dynamics of the free surface

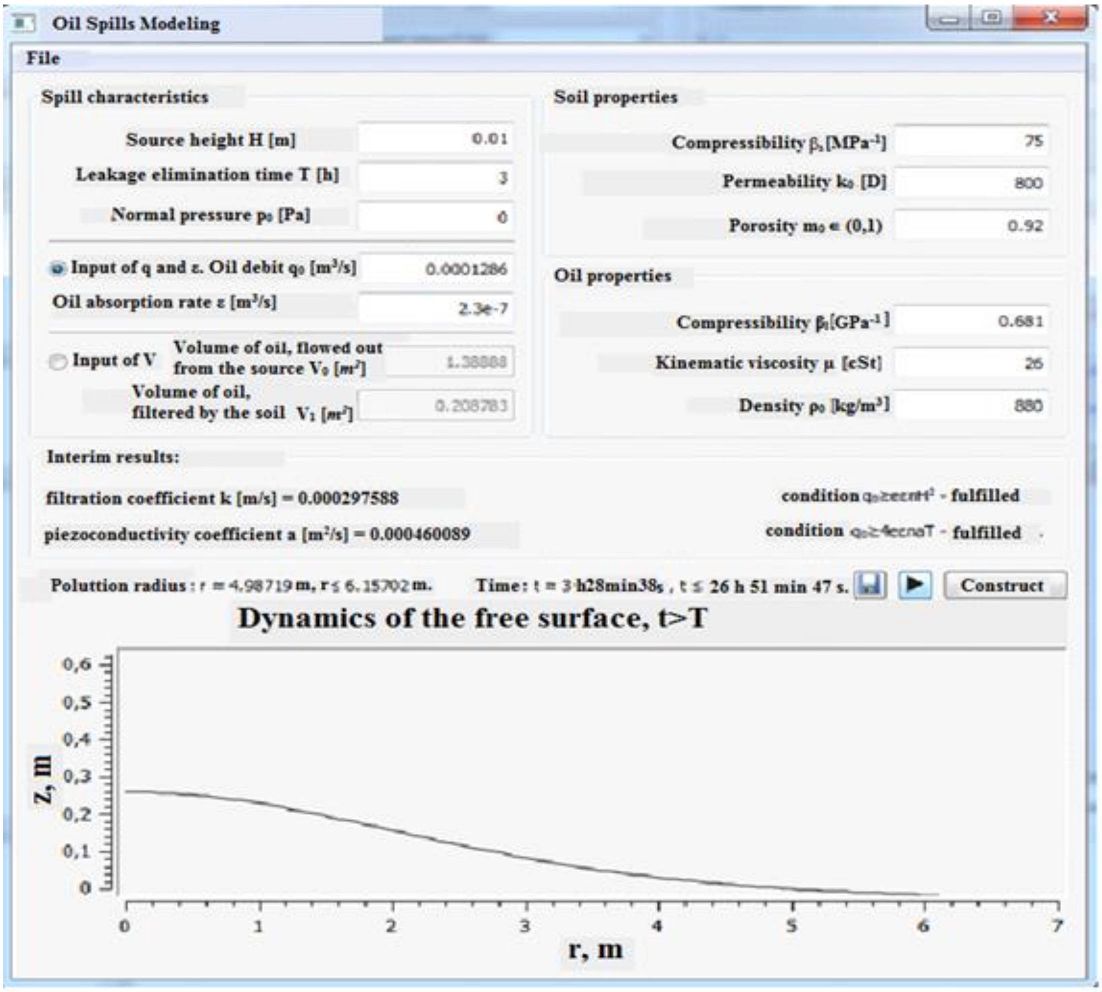


Fig. 3. The program interface

6) The calculation of piezoconductivity and filtration coefficients;

7) The model restrictions checking, and if yes, to step 8, otherwise to step 1;

8) Calculation of upper estimates;

9) Splitting the calculation periods, data calculation;

10) Calculation of approximate estimates;

11) Construction and export of the graph;

12) If the work is completed, then to step 13, otherwise to step 1;

13) The end.

There were implemented two variants of initial data input: the input of the production rate and rate of oil absorption by the soil, or input of volumes of the oil, flowed out from the source, and oil, filtered by the soil.

For all the scenarios we assume some common characteristics, namely: accidental spill occurs in the summer, at a temperature of 20 °C, in the tundra areas with peat soils. Peat has high porosity, up to 90–95 %, decreasing with the increase in the degree of decomposition, as well as a wide range of variation of the compressibility coefficient, from 1.5 to 80 MPa^{-1}, reaching the upper limit in heavily watered, poorly decomposed peats [5].

On this basis, for all four cases we take the porosity of peat equal to 0.92 and compressibility coefficient – 75 MPa^{-1}. The range of peat permeability coefficient variation has been calculated by using the filtration coefficient values [1, 6], and for calculations the value equal to 800 D was taken.

Such physical properties of crude oil as density and viscosity, in a case with a well, are taken from [7]. Compressibility coefficient, in its turn, is calculated using the empirical formula [8].

The height of the pipeline equals 2 m, and the height of the well – 0.01 m. In the scenarios "C" and "D" a pipeline rapture means there is a hole, which square is equal to the square of the pipeline cross section. For the scenario "D" the production rate (q_0) is calculated based on the known speed of oil in the pipeline and the pipeline radius.

Scenario "A".

As a result of an accident there occurred a gushing oil well. Repair of the well took 3 hours. Well gives heavy oil, its debit is 10 tons/day. The rate of absorption of oil by soil is $2.3\cdot10^{-7}$ m^3/s.

Scenario "B".

As a result of an accident there occurred a gushing oil well. Leak detection and repairs took three days. Well gives light fraction oil. During this time 250 t of oil flowed away, 50 t of which have been absorbed by the soil.

Scenario "C".

As a result of the pipeline rapture there occurred an accidental oil spill. The pipeline is located at a height of 2 m and transports light fraction oil. The time between the formation of the rapture and shutting of the flaps – 2 hours. During this time 50 t of oil flowed away, 11 t of which have been absorbed by the soil.

Scenario "D".

As a result of the pipeline rapture there occurred an accidental oil spill. The pipeline is located at a height of 2 m, has a diameter of 300 mm and transports heavy oil. Oil speed in the pipeline is 1 m/s. 1 hour passed before the flaps in the pipeline were shut. The rate of absorption of oil by the soil is $1.4\cdot10^{-4}$ m^3/s.

Table 1 shows the initial data prepared for the input into the program.

Table 1

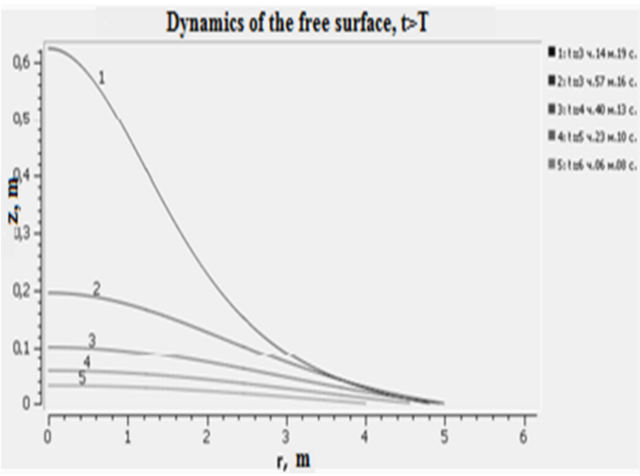
Initial data for calculations

Name	Symbol	Units of measurement	A	B	C	D
Spill characteristics						
Source height	H	*m*	0.01	0.01	2	2
Leakage elimination time	T	*h*	3	72	2	1
Oil debit (oil production rate)	q_0	m^3/s	$1.29 \cdot 10^{-4}$	–	–	0.07065
Oil absorption rate	ε	m^3/s	$2.3 \cdot 10^{-7}$	–	–	$1.4 \cdot 10^{-4}$
Volume of oil, flowed out from the source	V_0	m^3	–	304.878	60.975	–
Volume of oil, filtered by the soil	V_1	m^3	–	60.975	13.415	–
Soil properties						
Compressibility	β_s	MPa^{-1}	75			
Permeability	k_0	*D*	800			
Porosity	m_0	–	0.92			
Oil properties						
Compressibility	β_l	GPa^{-1}	0.681	0.822	0.822	0.705
Kinematic viscosity	μ	*cSt*	26	8	6	14
Density	ρ_0	kg/m^3	880	820	820	868

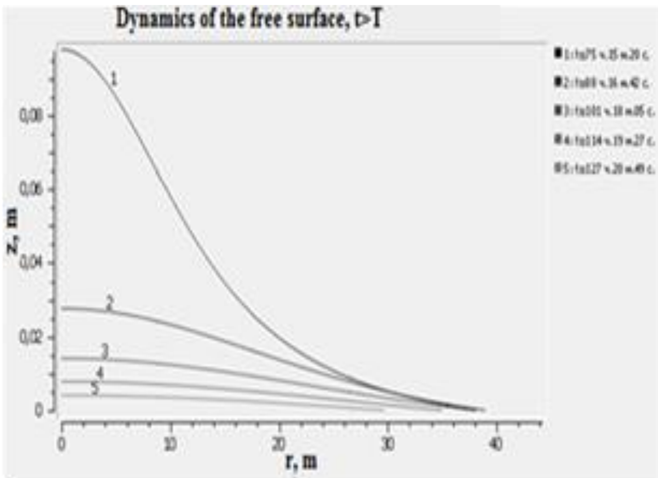
RESULTS OF THE RESEARCH

Table 2 shows the results of modeling for the four described scenarios. The upper estimates of the radius of pollution and time of flowing are calculated using the formulas: $r \le \sqrt{0.0678\frac{q_0}{\varepsilon} - H^2}$, $t \le \frac{q_0}{4a\pi\varepsilon}$. Based on the model restrictions, pollution area has a disc shape, that allows us to calculate its square.

Fig. 4 shows the dynamics of the free surface (z) over time, $t > T$.



1)



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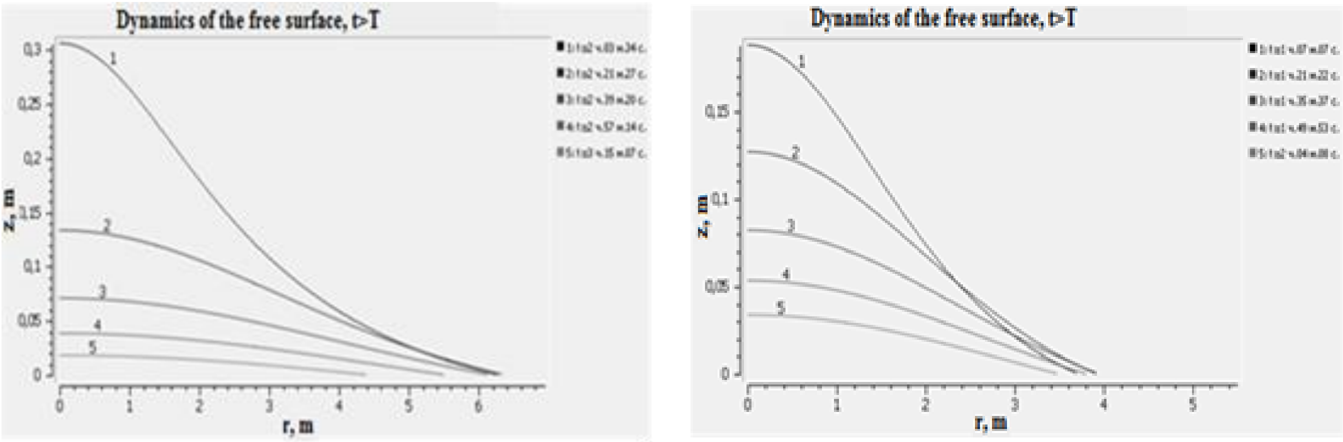

3) 4)

Fig. 4. Dynamics of the free surface. 1–4 – Scenarios A–D

Table 2

Results of modeling

Name	Symbol	Units of measurement	A	B	C	D
Volume of oil, flowed out from the source	V_0	m^3	1.38888	304.878	60.975	254.34
Volume of oil, filtered by the soil	V_1	m^3	0.20878	60.975	13.415	23.841
Radius of pollution	r	*m*	4.99	38.93	6.32	3.96
Upper limit of the pollution radius	r_{max}	*m*	6.16	44.24	6.95	5.5
Square of pollution	S	m^2	78.23	4761.23	125.49	49.27
Time	t	–	7 h 32 min	150 h 08 min	3 h 36 min	2 h 39 min
Upper limit of time	t_{max}	–	26 h 51 min	397 h 34 min	7 h 57 min	12 h 52 min

DISCUSSION OF OBTAINED RESULTS AND CONCLUSION

The paper studies the elastic filtration mode of oil in a layer based on the estimation of risks of environmental oil pollution because of accidental releases. We obtained upper and approximate estimates of the radius of the pollution and the duration of filtration of oil by soil. We have been determining the thickness of the oil film and the square of the spill during the formation of the zone of pollution. While conducting the research, preference was given to asymptotic methods, that required making assumptions about constant oil debit and horizontal surface of the soil. The obtained analytical solution satisfies few real situations, but has a simple calculation, therefore, the next step of the research is going to be the development of numerical solution, which allows us to consider arbitrary reliefs of the soil surface and the source function.

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МОДЕЛИРОВАНИЕ РАЗЛИВА НЕФТИ В АРКТИЧЕСКОЙ ЗОНЕ МОРСКОГО ПОБЕРЕЖЬЯ

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В работе исследуется упругий режим фильтрации нефти в грунте при аварии на нефтепроводе или нефтяной скважине, оцениваются масштабы загрязнения окружающей среды. Модель растекания и поглощения нефти грунтами строится на основе уравнения неразрывности и закона фильтрации Дарси, а также уравнений состояния, учитывающих сжимаемость среды от давления. Область фильтрации – это граница между загрязненным нефтью грунтом и воздухом. Область фильтрации нефти снизу ограничена поверхностью почвы, а сверху – свободной поверхностью, уравнение которой заранее неизвестно и подлежит определению. Рассматривается сценарий загрязнения почв от точечного источника, в качестве которого выступает место разрыва трубопровода или скважина. Получены предельные и приближенные оценки радиуса разлива нефти в зависимости от различных типов подстилающих поверхностей и характеристик нефти, а также условий окружающей среды. Рассчитана и приведена динамика свободной поверхности нефти в зависимости от радиуса растекания. Сделаны оценки временной продолжительности фильтрации нефти грунтом в зависимости от фракций нефти и типа почвы. Была рассчитана толщина пленки и площадь разлива нефти при условии постоянной скорости истекания нефти, горизонтальности подстилающей поверхности и близости давления в среде к нормальному. Для численной реализации построенной модели были рассмотрены различные сценарии разлива нефти. При заданных значениях температуры воздуха, пористости почвы и скорости фильтрации рассчитан радиус загрязнения в зависимости от фракции нефти, времени с момента аварии до момента устранения течи, а также скорости истечения нефти.

Ключевые слова: модель растекания и поглощения нефти грунтами, упругий режим фильтрации, область фильтрации, пористость почвы, радиус и площадь загрязнения, фракции нефти.

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